



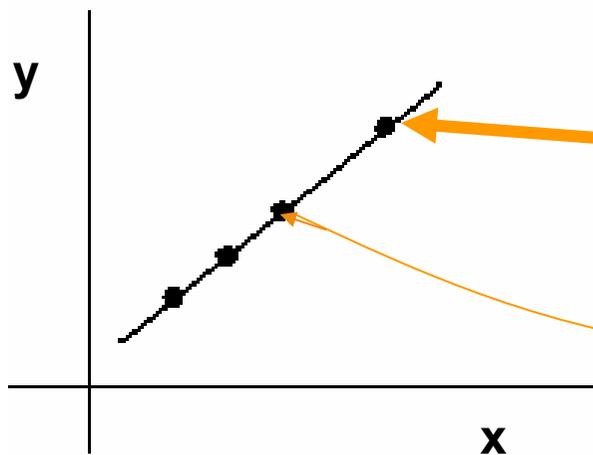
An approach for generic detection of conic equation from images

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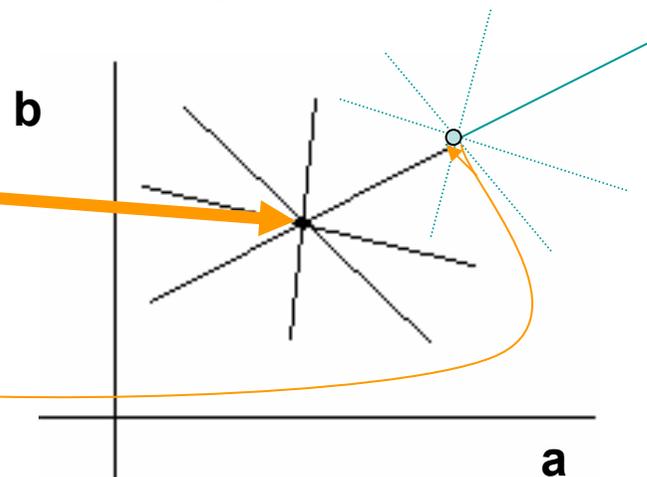
Hough Transform

- Hough transform is a technique to identify parameters of equations from forms in an image.
- Each point on a curve votes for several combination of parameters.

Image space: $y = ax + b$



Parameter space: $b = -ax + y$



Accumulator matrix

1- Form: straight line

Formula: $y = ax + b$

Image space => Parameter space

$$b = -ax + y$$

2- Discrete representation:

$x=1 \ y=1 \ a=0,1 \ b=0,9$

$x=1 \ y=1 \ a=0,2 \ b=0,8$

$x=1 \ y=1 \ a=0,3 \ b=0,7$

$x=2 \ y=1 \ a=0,1 \ b=0,8$

$x=2 \ y=1 \ a=0,2 \ b=0,6$

$x=2 \ y=1 \ a=0,3 \ b=0,4$

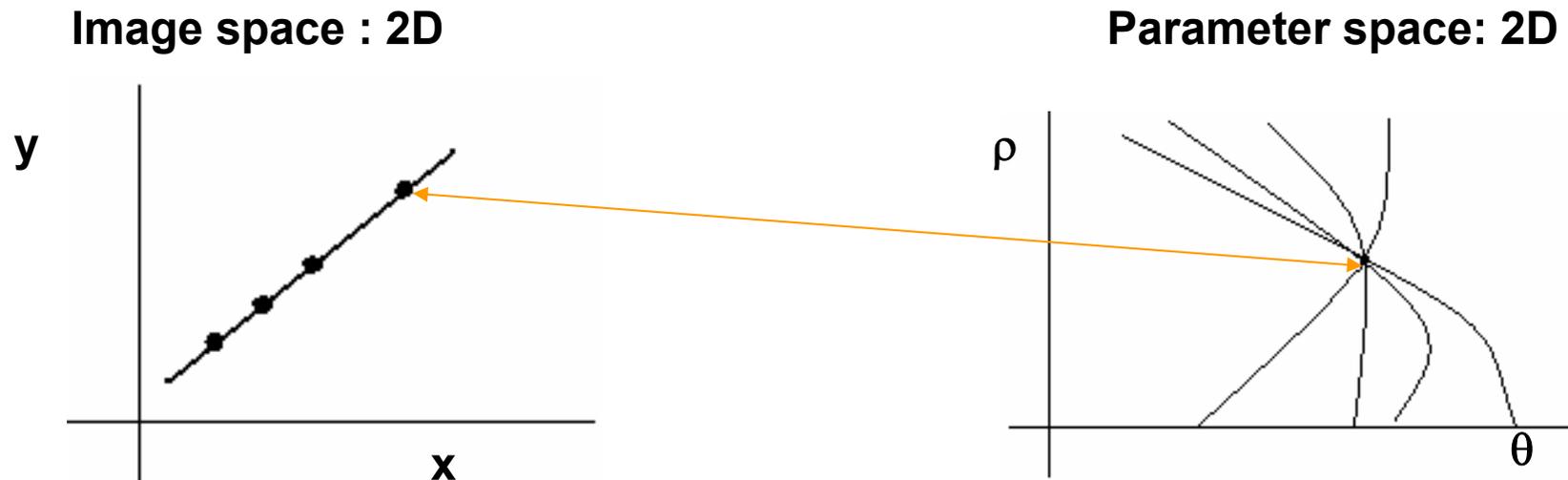
3- Bi-dimensional accumulator matrix

		a				
		0,1	0,2	0,3	0,4	0,5
b	0					
	0,2		10		60	
	0,4			10		
	0,6		2	8	120	30
	0,8	4	6			
1						

Elected straight line: $y = 0,4x + 0,6$

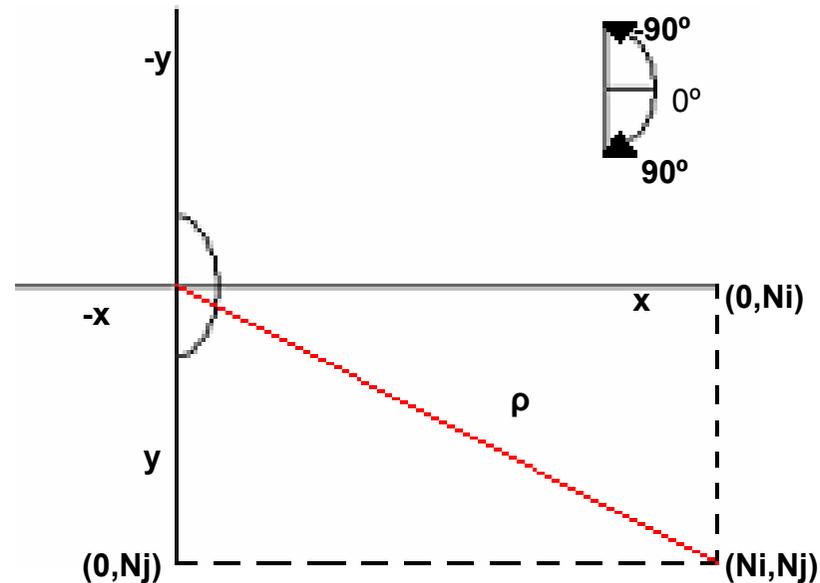
Hough Transform

- General equation: $b=y-ax$ $b=$ angular coefficient
- In practice, the polar form are used rather than explicit form to avoid problems with lines that are nearly vertical (b goes to infinity !).
- Polar system of coordinates: $\rho = x \cos(\theta) + y \sin(\theta)$



Ranges

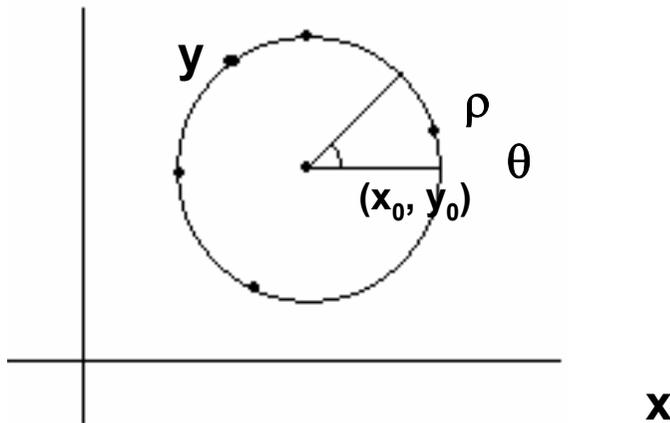
- **Matrix accumulator range:**
- $\rho \rightarrow -\sqrt{N_i^2 + N_j^2}$ to $\sqrt{N_i^2 + N_j^2}$
- $\theta \rightarrow -90^\circ < \theta \leq 90^\circ$



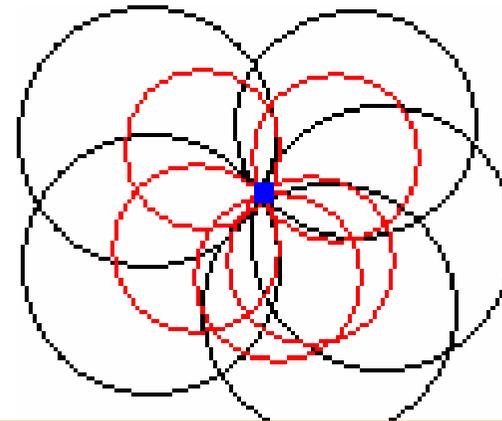
Detection of circular form

- General equation: $(x - x_0)^2 + (y - y_0)^2 = r^2 = \rho^2$
- Parametric equation Cartesian system of coordinates
- $x_0 = x - \rho \cos \theta$
- $y_0 = y - \rho \sin \theta$,
- where x_0 e y_0 are the coordinates of circle center.

Image space: 2D



Parameter space: 3D



Solution: technique perform changing ρ value in the possible range.

Parabolas

Parabola general equation on (ρ, β)
polar coordinates:

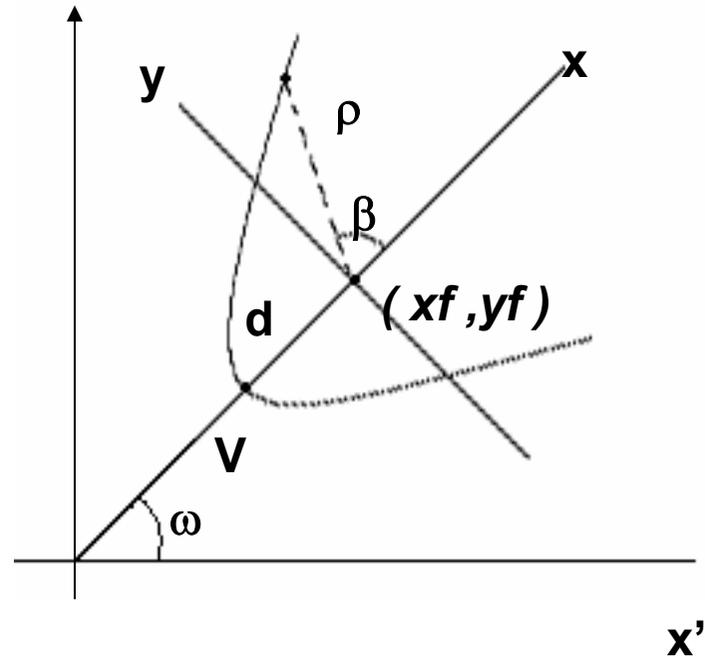
$$x' = x_f + \rho \cos \beta \cos \omega - \rho \sin \beta \sin \omega$$

$$y' = y_f + \rho \cos \beta \sin \omega + \rho \sin \beta \cos \omega$$

$$\rho = \frac{2d}{1 - \cos \beta} \quad \beta \neq 0$$

where: d is the distance between focus $F=(x_f, y_f)$ and vertex (V) ,

In this case is necessary to
obtain a 4-D accumulator
array $M(x_f, y_f, d, \omega)$.



Ellipses

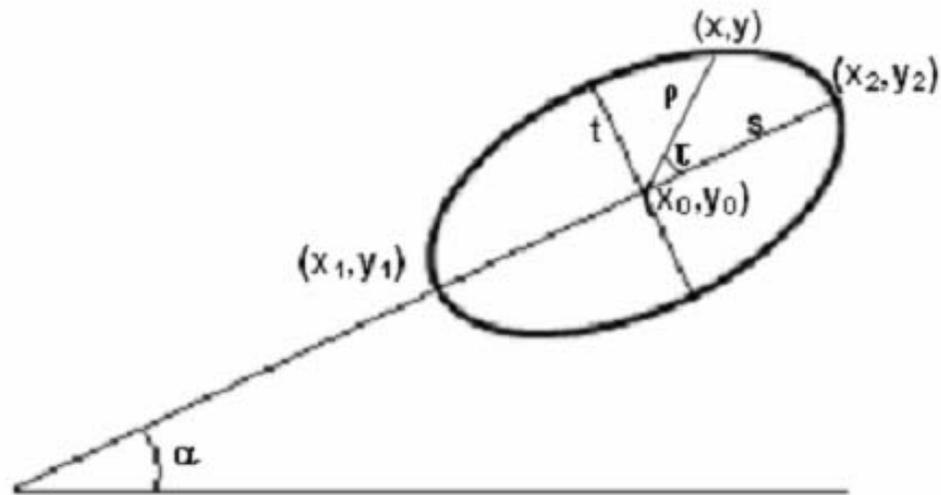
The Cartesian coordinates :

$$\frac{x^2}{s^2} + \frac{y^2}{t^2} = 1$$

where s is the biggest radius, t is the smallest radius.

(ρ, τ) are the polar coordinates, ρ is the distance between center and an edge point in the ellipse, and τ represents the angle between ρ and horizontal rotated axis x' .:

$$\rho^2 = \frac{s^2 t^2}{s^2 \sin^2 \tau + t^2 \cos^2 \tau}$$



$$x' = x_0 + s \cos \tau \cos \alpha + t \sin \tau \sin \alpha$$

$$y' = y_0 + t \sin \tau \cos \alpha - s \cos \tau \sin \alpha$$

Detecting ellipses without any adaptations is to cope with a 5-D accumulator array $M(x_0, y_0, s, t, \alpha)$.

Hyperbolas:

$$\frac{x^2}{m^2} - \frac{y^2}{n^2} = 1$$

A polar representation of hyperbole is:

$$\rho^2 = \frac{m^2 n^2}{m^2 \sin^2 \sigma - n^2 \cos^2 \sigma}$$

Wehe: the half biggest axis is m .

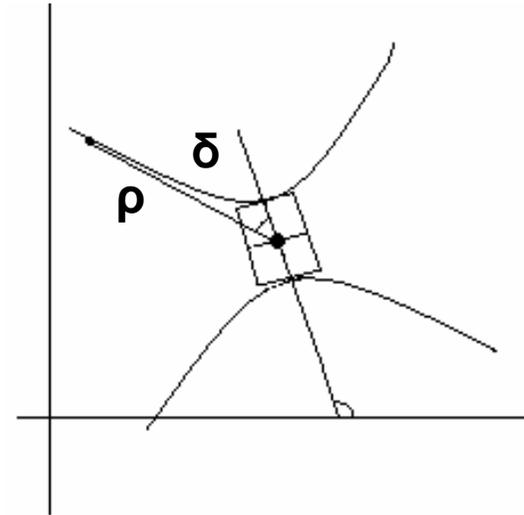
n is the half of smallest axis in hyperbole,

ρ is the distance between centre and an edge point in the hyperbole

δ represents the angle between ρ and m axis.

(ρ, δ) are the polar coordinates: $x' = x_0 + m \cos \delta \cos \sigma - n \sin \delta \sin \sigma$

$$y' = y_0 + n \sin \delta \cos \sigma + m \cos \delta \sin \sigma$$



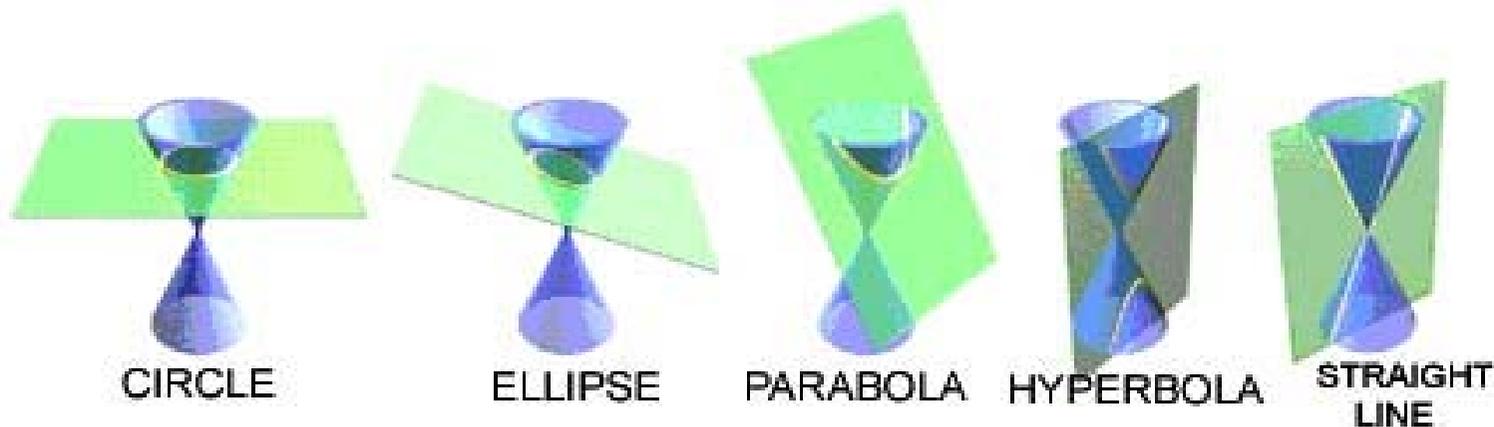
For Hough transform a 5-D accumulator array

$M(x_0, y_0, m, n, \sigma)$ is necessary

Proposed unified approach

- **This methodology intends to unify all type of Hough detection based on each conic features.**
- **From a minimum information it is possible reconstruct a form.**
- **Hand drawing can also be performed through this method.**

Conics



- **Identify a known form**
- Start the search for the polar coordinates (polar radius, polar angle) of each image pixel,
- Considers range of values involves the election of **each conic detection**

Conic parameters

Set of parameters $P = (p_1, p_2, p_3, \dots)$.

Lower limits $L = (l_1, l_2, l_3, \dots)$ Upper limits $U = (u_1, u_2, u_3, \dots)$

Example:

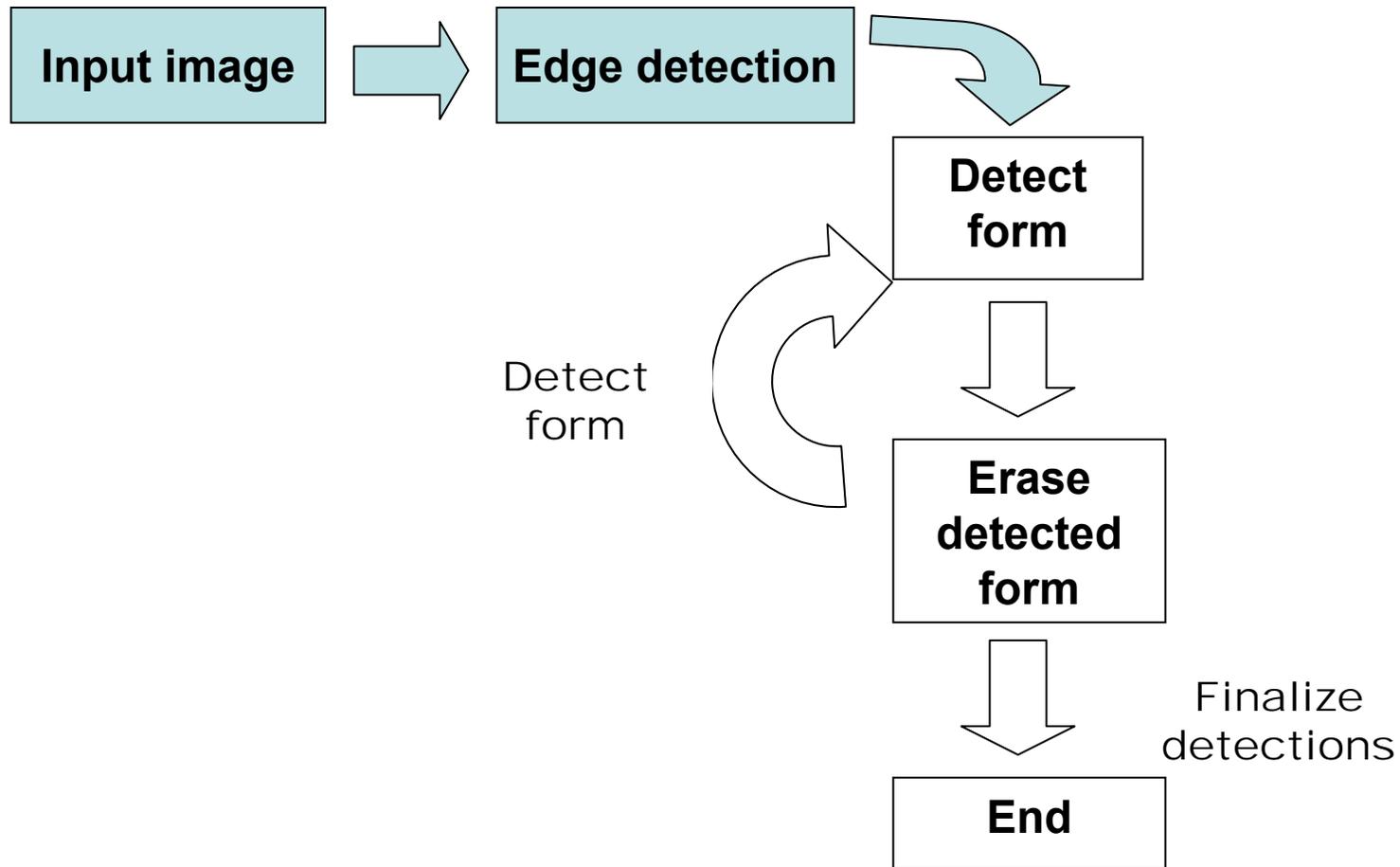
Parabola $P = (x_f, y_f, d, \omega)$

Lower limits $L = (0, 0, \min, 0)$ Upper limits $U = (N_i, N_j, \max, 2\pi)$

Accumulator matrix dimension:

$$D = \begin{cases} \text{length}(P) + 1 & \text{if the conic has rotation} \\ \text{length}(P) & \text{otherwise} \end{cases}$$

Detection scheme for complex images



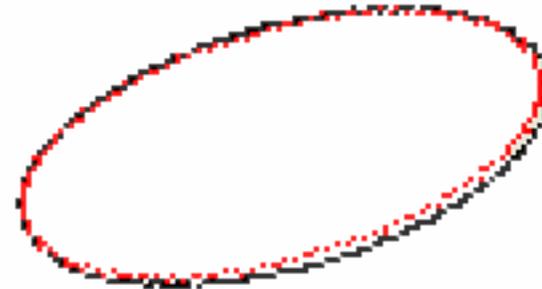
Conics detection in whatever position

Experiments

Input image



Detected form



(x_0, y_0)	s	t	α	Tempo de execução
(82,92)	51	21	16	0,078s

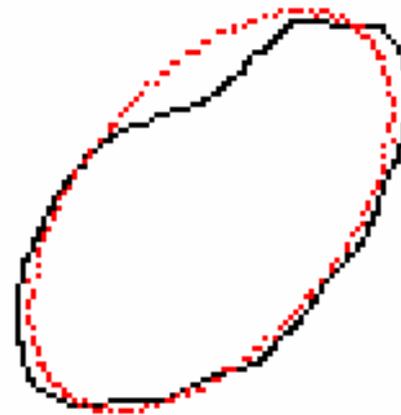
Approaching conic equation from Hand drawing form

Experiments

Hand drawing form



Ellipse detected form

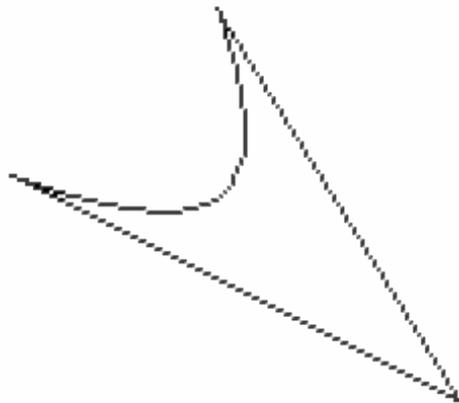


(x_0, y_0)	s	t	α
(67,55)	46	26	50°

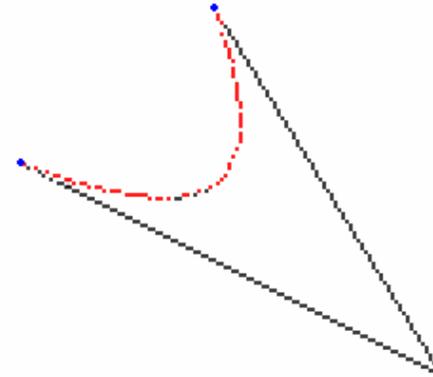
Multiple Generic Form in the same image

The figure below consist of two straight lines and a parabola.

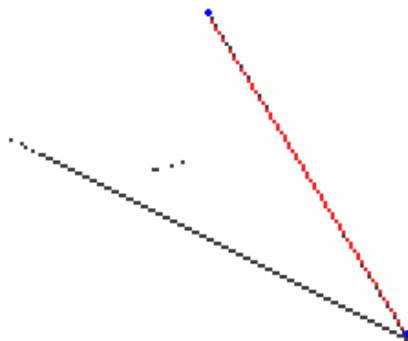
Input image



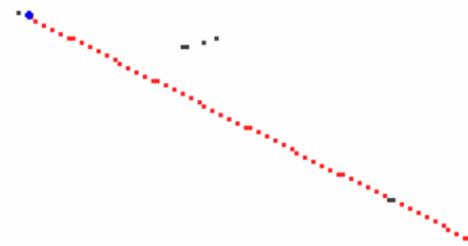
Detected parabola



First detected straight line



Second detected straight line



Other Applications

Brinell Hardness test for metallographic images

Input Image

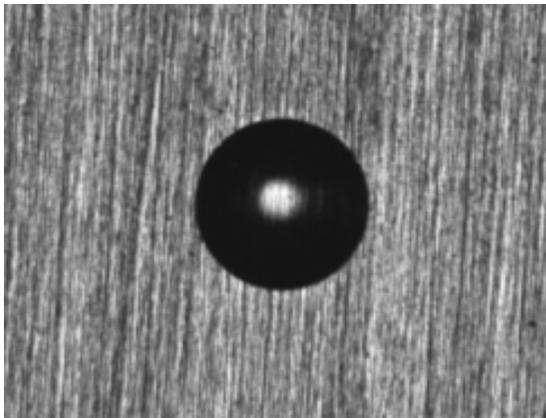
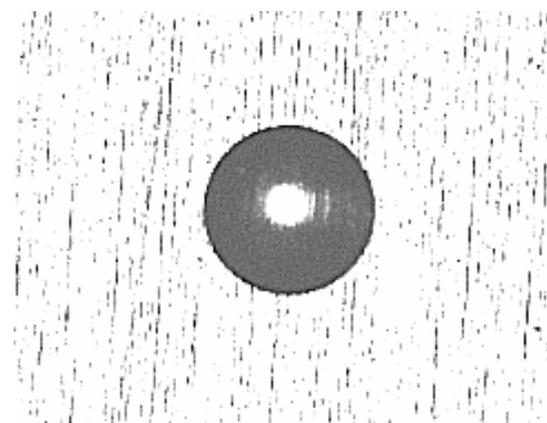
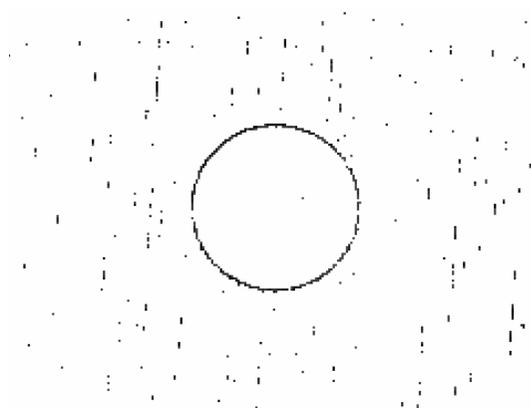


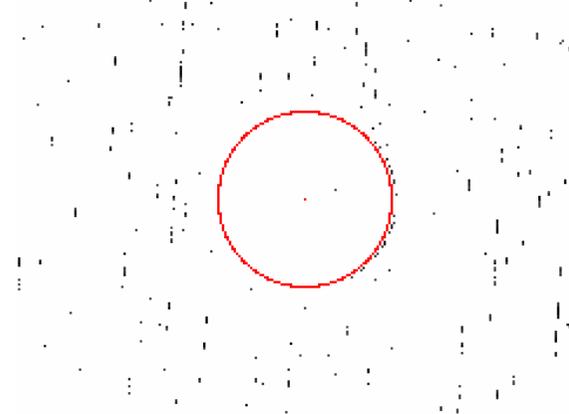
Image after a filter



After threshold



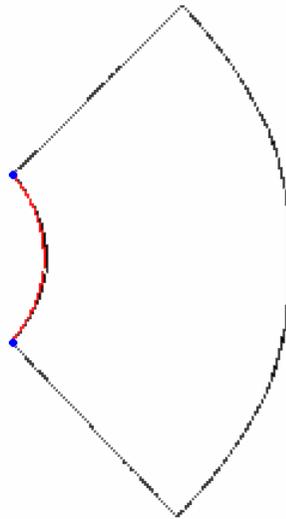
Detected circle



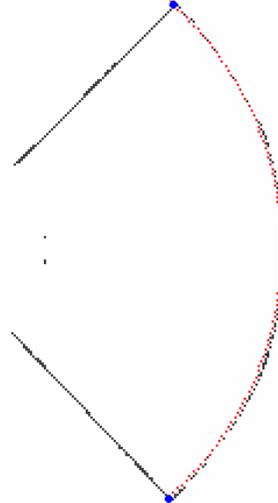
Equation of patterns for apparel industry

The figure below consist of two straight lines and two circle of arcs.

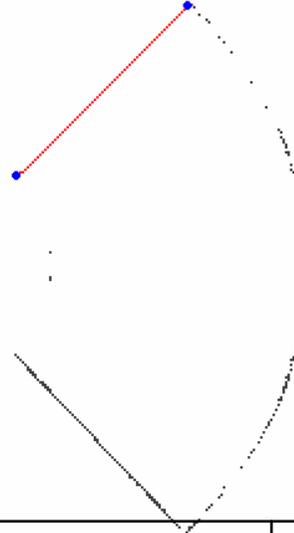
First Circle Arc



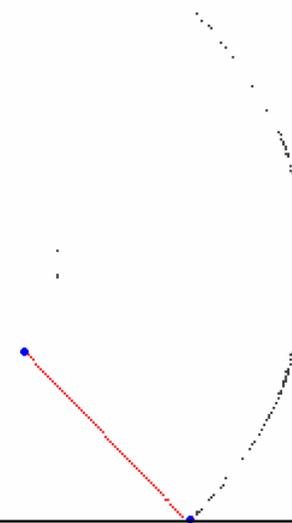
Second Circle Arc



First Straight Line



Second Straight Line



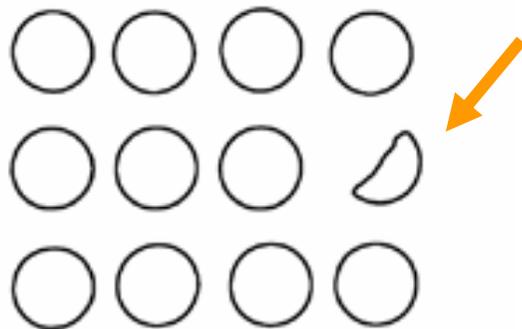
Arc	x_0	y_0	ρ	Initial	Final
		109	32	51	(74,70)
	109	46	140	(5,140)	(216,136)
Lines	θ	ρ	-	Initial	Final
	45	102	-	(5,141)	(73,71)
	135	53	-	(145,70)	(215,140)

Finding radius variations and incomplete form

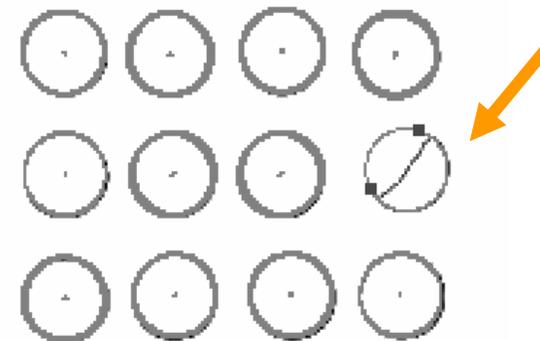
Input image of medicines, one of them has a fail and is detected as arch.



Pre-processed Image



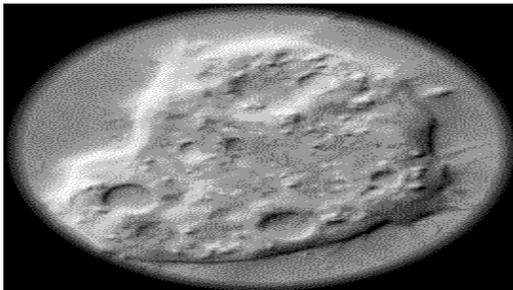
▪ **Detected image**



Conic equation identification using few points

Experiments

Input image
Image



Failed Edge Detected



Detected form



(x_0, y_0)	s	t	α	Time
(70,94)	62	30	0	0,047s

Conclusion

- Conic detections (as open or closed curves)
- Generic equation parameters identification (also initial and final points).
- Accumulator matrix dimension
- Features extraction
- Store for rebuild
- Multiples application from edges detected images



An approach for generic detection of conic form

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