From subexponentials in linear logic to concurrent constraint programming and back

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October 13, 2015
Workshop on Logic, Language and Information

Motivation

Our objetive

Languages and reasoning techniques for the specification and verification of concurrent systems where different modalities can be combined.

Potential target applications:

- Multimedia Interactive Systems
- Biochemical Systems
- Mobile systems, Social Networks, distributed systems.

- Spatial modalities: locations, places, devices, biochemical interaction domains....
- Epistemic modalities: beliefs, opinions, facts, lies...
- Temporal modalities: System's configuration evolves along time-units.

Motivation

Concurrent Constraint Programming (CCP)

A simple and powerful model of concurrency tied to logic:

- Systems are specified by constraints (i.e., formulas in logic) representing partial information on the variables of the system.
- Agents tell and ask constraints on a shared store of constraints.

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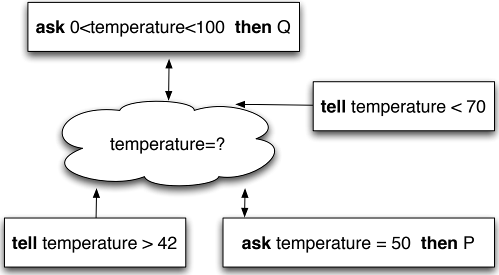

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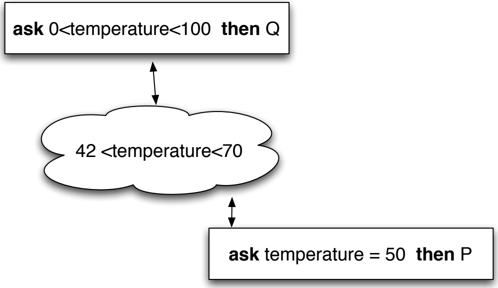

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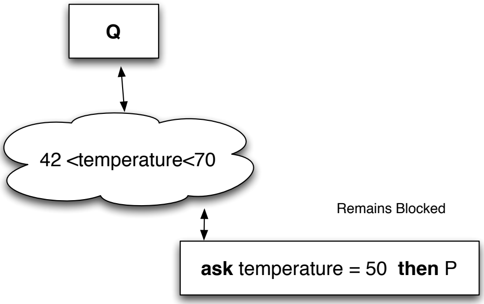

Motivation
CCP Calculi

CCP has been extended to deal with different application domains:

- tcc: Reactive and timed systems [SJG94].
- pccp: Probabilistic choices [GJS97].
- lccp: Linearity and resources [FRS01].
- ntcc: Time, non-determinsim and asynchrony [NPV02].
- cc-pi, utcc: Mobility [BM07, OV08].
- soft-ccp : Soft constraints and preferences [BMR06].
- eccp and sccp: Epistemic and Spatial reasoning [KPPV12].

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The idea

Reason about different CCP systems in one logical framework.

Motivation

Subexponentials in Linear Logic (SELL)

Linear logic:

- Formulas are seen as resources, e.g., $c \otimes c \not\vdash c$.
- Classical reasoning is recovered by the use of exponentials: $!c \vdash c \otimes c$

Subexponentials [DJS93]

Intuitively, $!^a F$ means "F holds in location a". Such exponentials are not canonical:

$$\text{(in general) } !^a c \not\equiv !^b c \text{ if } a \neq b$$

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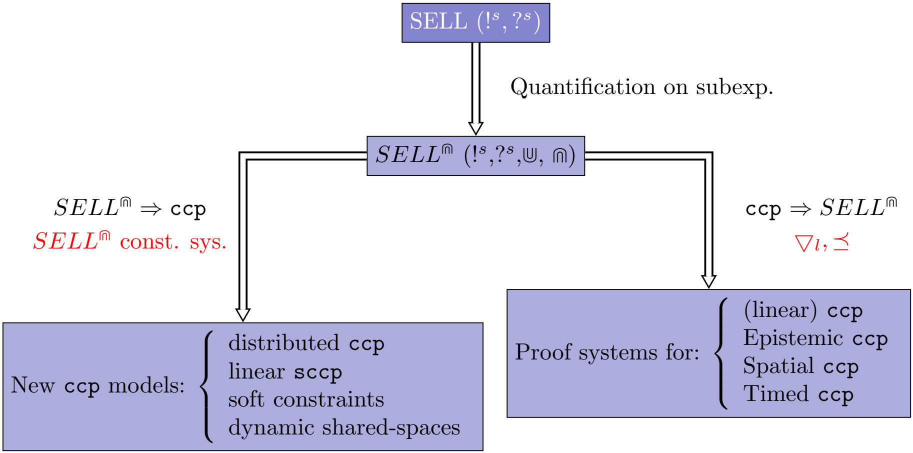
The Idea

Quantification on location may allow the specification of interesting behaviours in concurrency.

This work is about



Outline

CCP: The language of Processes

Concurrent Constraint Programming

- tell(c) adds c to the store (d) leading to $d \wedge c$.
- The process **ask** c **then** P evolves into P if c can be deduced from the store. This is a simple and powerful synchronization mechanism.
- $P \parallel Q$: parallel execution of P and Q.
- $(\textbf{local}\, x)\, P$: local variables.
- Given a definition, $p(\overline{y}) \stackrel{\text{def}}{=} P$, the process $p(\overline{x})$ reduces to $P[\overline{x}/\overline{y}]$.

A simple example: Classical coffee machine

$$(\textbf{tell}(coin) \parallel \textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee), \texttt{true}) \longrightarrow$$
$$(\textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee), coin) \longrightarrow$$
$$(\textbf{skip}, \underline{coin \wedge coffee})$$

Outline

Linear CCP [FRS01]

Constraints as formulae in (a fragment of) Girard's ILL:

Ask agents consume information when evolving.

The linear coffee machine

$$(\textbf{tell}(coin) \parallel \textbf{ask}\ coin\ \textbf{then}\ coffee, \texttt{true}) \longrightarrow$$
$$(\textbf{ask}\ coin\ \textbf{then}\ coffee, coin) \longrightarrow (\textbf{skip}, \underline{coffee})$$

Declarative Reading of lcc processes

[FRS01] showed that (L)CCP processes can be read as formulae in ILL:

$$(P, c) \longrightarrow^{*} (Q, d) \text{ iff } \mathcal{L}[\![P]\!] \otimes c \vdash \mathcal{L}[\![Q]\!] \otimes d$$

Focusing and Adequacy

Logical and operational steps do not correspond (closely) to each other:

- Process: $P = \textbf{tell}(c) \parallel \textbf{ask}\ c\ \textbf{then}\ \textbf{tell}(d) \parallel \textbf{ask}\ d\ \textbf{then}\ \textbf{tell}(e)$
- Operational side: $P \Downarrow_e$ (P outpus e).
- Logical side $c \otimes (c \multimap d) \otimes (d \multimap e) \vdash e$, but:

$$\dfrac{\dfrac{}{e \vdash e} \quad \dfrac{\dfrac{}{c \vdash c} \quad \dfrac{}{d \vdash d}}{c, c \multimap d \vdash d}}{c, c \multimap d, d \multimap e \vdash e}$$

Andreoli's focusing system [And92]:

- negative connectives $\multimap, \&, \top, \forall, \ldots$
- positive connectives: $\otimes, \oplus, \exists, \ldots.$

Focusing and Adequacy

Negative Phase

$$\frac{[\mathcal{K}:\Gamma],\Delta,F,G\longrightarrow\mathcal{R}}{[\mathcal{K}:\Gamma],\Delta,F\otimes G\longrightarrow\mathcal{R}}\otimes_L \quad \frac{[\mathcal{K}:\Gamma],\Delta,F\longrightarrow G}{[\mathcal{K}:\Gamma],\Delta\longrightarrow F\multimap G}\multimap_R \quad \frac{[\mathcal{K}:\Gamma],\Delta\longrightarrow G[x_e/x]}{[\mathcal{K}:\Gamma],\Delta\longrightarrow\forall x.G}\forall_R$$

Positive Phase

$$\frac{[\mathcal{K}_1:\Gamma_1]\!-_F\!\!\to \quad [\mathcal{K}_2:\Gamma_2]\!-_G\!\!\to}{[\mathcal{K}_1\otimes\mathcal{K}_2:\Gamma_1,\Gamma_2]\!-_{F\otimes G}\!\!\to}\otimes_R \quad \frac{[\mathcal{K}_1:\Gamma_1]\!-_F\!\!\to \quad [\mathcal{K}_2:\Gamma_2]\xrightarrow{H} G}{[\mathcal{K}_1\otimes\mathcal{K}_2:\Gamma_1,\Gamma_2]\xrightarrow{F\multimap H} G}\multimap_L$$

If we decide to focus on $c \otimes (c \multimap d) \otimes (d \multimap e) \vdash e$, the atom d must be already in the context!

Declarative Reading of `lcc` processes [OP15]

Focused proofs corresponds, one-to-one, to operational steps in (I)CCP.

$$(P,c)\longrightarrow^{*}(Q,d) \text{ iff } \mathcal{L}[\![P]\!]\otimes c\vdash\mathcal{L}[\![Q]\!]\otimes d$$

Modalities in CCP

Epistemic and Spatial behavior in CCP

Assume a set of agents A={i,j,k...},

- $[P]_i$ means P runs in the space-agent i.
- $s_i(c)$ means the constraint (information) c holds for agent i.

Constraints are of the form $s_i(c)$. Two possible interpretations:

1. Epistemic:
   - $s_i(c)$: i knows c (and then, c is true).
   - $s_j(s_i(c))$: j knows that i knows c (and then, j knows c).
2. Spatial
   - $s_i(c)$: c holds in the space of i.
   - $s_j(s_i(c))$: c holds in the space that j conferred to i but c does not necessarily hold in j.

Epistemic CCP

Some properties for s_i:

1. $s_i(c) \vdash_{\Delta_e} c$ (believes are facts)
2. $s_i(s_i(c)) = s_i(c)$ (idempotence)

In eccp, knowledge of agents becomes a fact and information propagates to outermost spaces:

$$
\begin{array}{l}
(\textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee) \parallel [\textbf{tell}(coin)]_i, \texttt{true}) \longrightarrow \\
(\textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee) \parallel, s_i(coin)) \longrightarrow \\
(\textbf{tell}(coffee), s_i(coin)) \longrightarrow \\
(\textbf{skip}, \underline{s_i(coin) \wedge coffee})
\end{array}
$$

Spatial CCP (Information Confinment)

In sccp, inconsistency (and information) is confined:

1. $s_i(0) \not\vdash_{\Delta_s} s_j(0)$ (false is not propagated outside locations).
2. $s_i(0) \not\vdash_{\Delta_s} 0$ (falsity is not global)

$$(\textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee) \parallel [\textbf{tell}(coin)]_i, \texttt{true}) \longrightarrow$$
$$(\textbf{ask}\ coin\ \textbf{then}\ \textbf{tell}(coffee), \underline{s_i(coin)}) \not\longrightarrow$$

How to give a declarative interpretation of such modalities ?

Outline

Subexponentials [DJS93] in Linear Logic

Subexponential Signature

$\Sigma = \langle I, \preceq, U \rangle$ where I is a set of labels, $U \subseteq I$ set of unbounded subexp and $\preceq$ is a pre-order among the elements of I.

$$\frac{\Gamma, F \longrightarrow G}{\Gamma, !^a F \longrightarrow G}\ !^a{}_L \qquad \frac{!^{a_1} F_1, \ldots, !^{a_n} F_n \longrightarrow F}{!^{a_1} F_1, \ldots, !^{a_n} F_n \longrightarrow !^a F}\ !^a{}_R, \text{ provided } a \preceq a_i$$

$$\frac{\Gamma \longrightarrow G}{\Gamma, !^b F \longrightarrow G}\ W \qquad \frac{\Gamma, !^b F, !^b F \longrightarrow G}{\Gamma, !^b F \longrightarrow G}\ C$$

Assume now two separated rooms a and b, i.e., $a \not\preceq b$ and $b \not\preceq a$.

$$(!^a coin \multimap !^a coffee) \otimes !^b coin \not\vdash !^b coffee$$

- What about a specification like $\forall l.(!^l coin \multimap !^l coffee)$?
- We need a theory for existential/universal quantification on subexponentials.

Quantification on Locations [NOP13]

$$\frac{\mathcal{A}; \mathcal{L}; \Gamma, P[l/x] \vdash G}{\mathcal{A}; \mathcal{L}; \Gamma, \Cap x : a.P \vdash G} \Cap_L \qquad \frac{\mathcal{A}, l_e : a; \mathcal{L}; \Gamma \vdash P[l_e/x]}{\mathcal{A}; \mathcal{L}; \Gamma \vdash \Cap x : a.P} \Cap_R$$

$$\frac{\mathcal{A}, l_e : a; \mathcal{L}; \Gamma, P[l_e/x] \vdash G}{\mathcal{A}; \mathcal{L}; \Gamma, \Cup x : a.P \vdash G} \Cup_L \qquad \frac{\mathcal{A}; \mathcal{L}; \Gamma \vdash P[l/x]}{\mathcal{A}; \mathcal{L}; \Gamma \vdash \Cup x : a.P} \Cup_R$$

- Creating "new" locations: $\Gamma, \Cup l.(F) \vdash G$
- Asserting something about all locations: $\Gamma, \Cap l.(F) \vdash G$
- Proving that all locations satisfies G: $\Gamma \vdash \Cap l.(G)$
- Proving that G holds in some location: $\Gamma \vdash \Cup l.(G)$

Theorem (Cut-elimination) [NOP13]

For any signature Σ, the proof system $\mathrm{SELL}^{\Cap}$ admits cut-elimination.

Epistemic and Spatial Encodings

The intuition

Connective	Meaning
$\nabla_s = !^s$	$!^s P$ is located at s.
$\nabla_s = !^s ?^s$	$!^s ?^s P$ is confined to s.
$\Cap l : a\ P$	P can move to locations below (outside) a

Epistemic Modalities

$\preceq$	Meaning
$a.a \sim a$	Modalities are idempotent: $[[P]_a]_a \sim [P]_a$
$a \preceq a.b$	Processes move outside $[[P]_b]_a \longrightarrow [P \parallel [P]_b]_a$

Spatial Modalities

$\preceq$	Meaning
$a \not\preceq b$	P does not communicate with Q in $[P]_a \parallel [Q]_b$
$a.a \not\sim a$	Modalities are not necessarily idempotent.
$a \not\preceq a.b$	Processes are confined: $[[P]_b]_a \not\sim [P \parallel [P]_b]_a$

Adequacy

Take for instance:

$$\mathcal{P}[\![\mathbf{tell}(c)]\!]_a = !^{\mathfrak{p}(a)} \Cap s : a.(\mathcal{C}[\![c]\!]_s)$$

We get the following (focused) derivation in $\mathrm{SELL}^{\Cap}$:

$$\dfrac{\dfrac{\dfrac{[\mathcal{C}', \mathcal{D}, \mathcal{P}] \longrightarrow [G]}{[\mathcal{C}, \mathcal{D}, \mathcal{P}], \mathcal{C}[\![c]\!]_s \longrightarrow [G]}\; n \times \exists_l, m \times \otimes_l, j \times !_l}{[\mathcal{C}, \mathcal{D}, \mathcal{P}] \xrightarrow{\Cap s : a.\mathcal{C}[\![c]\!]_s} [G]}\; \Cap_L, R_l}{[\mathcal{C}, \mathcal{D}, \mathcal{P} +_{\mathfrak{p}(a)} \Cap s : a.\mathcal{C}[\![c]\!]_s] \longrightarrow [G]}\; D$$

Theorem (Adequacy)

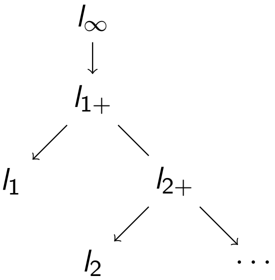
Let P be an eccp/sccp process, then,

$$P \Downarrow_c \textit{ iff } \mathcal{P}[\![P]\!] \longrightarrow \mathcal{C}[\![c]\!]_{nil}$$

Timed Modalities in *SELLF*

The tcc calculus

$$P, Q, \ldots := \textbf{tell}(c) \ldots \mid \circ P \mid \Box P$$



$$\begin{array}{lcl} \mathcal{P}[\![c]\!]_l & = & \nabla_{\bar{l}}\, c = !^{l} ?^{l} c \\ \mathcal{P}[\![\circ P]\!]_i & = & \mathcal{P}[\![P]\!]_{i+1} \\ \mathcal{P}[\![\Box\, P]\!]_i & = & !^{\mathfrak{p}(\infty)} \Cap l : i+ (\mathcal{P}[\![P]\!]_l) \end{array}$$

Theorem (Adequacy)

Let P be a timed process, $(\mathcal{C}_t, \Delta_t)$ be a CS. Then $P \Downarrow_c$ iff $!^{\mathfrak{c}(\infty)}[\![\Delta_t]\!], \mathcal{P}[\![P]\!]_1 \longrightarrow \Cup l : 1+ . !^{\mathfrak{c}(l)} ?^{\mathfrak{c}(l)} c \otimes \top$.

Outline

Subexponential CCP

From $\mathrm{SELL}^{\Cap}$ to CCP

Assume a constraint system where subexponentials are allowed:

$$F \quad := \quad 1 \mid A \mid F \otimes F \mid \exists \overline{x}.F \mid !^{a} F \mid !^{s} ?^{s} F$$

- $!^{a} c = (\!| c |\!)_{a}$: c holds (is believed) with preference a.
- $!^{s} ?^{s'} c = [c]_{s'}^{s}$: c holds in any space in the hierarchy $s' : s$.

Processes are allowed to create and communicate locations:

$$P, Q \quad := \quad \mathbf{tell}(c) \mid (\mathbf{local}\, \overline{x}; \overline{l})\, Q \mid (\mathbf{abs}\ \overline{x}; \overline{l}; c)\, Q \mid P \parallel Q \mid [P]_{s}$$

What do we get?

A declarative model for concurrency where different modalities can be combined!

Programming Examples

Sharing Information

Assume that $s'' \preceq s' \preceq s$:

1. $[c]^{s}_{s'} \vdash_\Delta [c]_{s'}$ (information c can be propagated to the inner/lower space s');
2. $[c]^{s}_{s''} \vdash_\Delta [c]_{s'}$ (information c can be propagated to the intermediate location in the hierarchy);
3. $[c]_s \nvdash_\Delta [c]^{s}_{s'}$ (information is confined if sharing is not explicit);

Example (Agent 86's Coffee Machine)

$$(\textbf{local}\ l : m/c, l' : m/c)\,\textbf{tell}([coin]_l) \parallel \textbf{ask}\ [coin]_l\ \textbf{then}\ \textbf{tell}([coffee]_{l'})$$

Example (Nested Locations)

$$(\textbf{local}\ l : m/c, l' : l)\,\textbf{tell}([coin]_l) \parallel \textbf{ask}\ [coin]_l\ \textbf{then}\ \textbf{tell}([coffee]_{l'})$$

Programming Examples

Temporal and Spatial Dependencies

Example

$[[c]_2]_{s_a} \otimes [[d]_{3+}]_{s_{a'}}$ means that c holds for agent a in time-unit 2 while d holds for a' in all future time-unit $t \geq 3$. This is useful for describing sets of biochemical reactions ([CFHO15]).

Mobility

$$\text{for names:} \quad \exists x.P \wedge \forall y.Q \quad \rightsquigarrow \quad \exists x.(P \wedge Q)$$
$$\text{for locations:} \quad \Cup l.\, \nabla_l P \wedge \Cap w.\, \nabla_w Q \quad \rightsquigarrow \quad \Cup l.(\nabla_l P \wedge \nabla_l Q)$$

Example (Service Oriented Computing)

$$\textbf{request}(a,b) \stackrel{\text{def}}{=} (\textbf{local}\, x, l : \{a,b\})\,(\textbf{tell}([\mathtt{com}(x)]_b) \parallel \textbf{ask}\ [\mathtt{com}(x)]_a\ \textbf{then}\ (\textbf{tell}([\mathtt{com}(x)]_l) \parallel P))$$
$$\textbf{accept}(a,b) \stackrel{\text{def}}{=} (\textbf{abs}\ y : b; [\mathtt{com}(y)]_b)\,(\textbf{tell}([\mathtt{com}(y)]_a) \parallel (\textbf{abs}\ k : b; [\mathtt{com}(y)]_k)\ Q)$$

Preferences and Soft Constraints

Using a c-semiring as a subexponential signature, agents can tell/ask preferences:

Examples of c-semirings $\langle \mathcal{A}, +, \times, \bot_A, \top_A \rangle$

- Fuzzy: $S_F = \langle [0,1], max, min, 0, 1 \rangle$ – Preferences
- Probabilistic: $S_P = \langle [0,1], max, \times, 0, 1 \rangle$
- Weighted: $S_w = \langle \mathcal{R}^-, max, +, -\infty, 0 \rangle$ – Costs

SELLS System [PON14], Promotion Rule

$$\frac{!^{a_1} F_1, \cdots, !^{a_n} F_n \longrightarrow G}{!^{a_1} F_1, \cdots, !^{a_n} F_n \longrightarrow !^b G} \; b \preceq a_1 \times \ldots \times a_n$$

1. Fuzzy: $(\!|c|\!)_{0.7} \vdash_\Delta (\!|c|\!)_{0.5}$ (if c is added with a higher preference a', then it can be deduced with a lower preference a);
2. Probabilistic: $(\!|c|\!)_{0.7} \otimes (\!|d|\!)_{0.3} \vdash_\Delta (\!|c \otimes d|\!)_a$ ($a \leq 0.21$).

Outline

Concluding Remarks

- We showed that subexponentials can express interesting behaviors in concurrency.
- The resulting system turned out to be a nice proof system for different flavors of CCP:
  - Spatial modalities, where nested locations can be dynamically created and shared.
  - Knowledge
  - Temporal Modalities
  - Soft constraints and preferences
- The logical system guided the design for new (still declarative) constructors for CCP.
- Two concrete applications so far: logic/CCP semantics for:
  - *P*-Systems.
  - Reactive Scores.

Thank you!

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