Polluted Resolution and other Combined Proof Search Methods for Propositional Modal Logics

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WOLLI, 2015

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A Modal-Layered Resolution Calculus for K - **Tableaux 2015**

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Motivation

- K_n, the smallest multi-modal normal logic, extends propositional logic with a fixed, finite set of modal operators.
- Formally, the *set of well-formed formulae*, WFF_{K_n}, is the least set such that:
  - $p \in \mathcal{P} = \{p, q, p', q', p_1, q_1, \ldots\}$ and true are in WFF_{K_n};
  - if φ and ψ are in WFF_{K_n}, then so are $\neg\varphi$, $(\varphi \land \psi)$, and $\boxed{a}\varphi$ for each $a \in \mathcal{A}_n = \{1, \ldots, n\}$.
- Formulae are interpreted, as usual, with respect to Kripke structures:

$$\langle \mathcal{W}, w_0, \mathcal{R}_1, \ldots, \mathcal{R}_n, \pi \rangle$$

where

$\langle \mathcal{M}, w \rangle \models \boxed{a}\varphi$ if, and only if, for all w', $w\mathcal{R}_a w'$ implies $\langle \mathcal{M}, w' \rangle \models \varphi$.
- Abbreviations: false $=$ $\neg$true, $(\varphi \lor \psi) = \neg(\neg\varphi \land \neg\psi)$, $(\varphi \to \psi) = (\neg\varphi \lor \psi)$, and $\Diamond_a\varphi = \neg\boxed{a}\neg\varphi$.

Reasoning Tasks

$$\langle \mathcal{W}, w_0, \mathcal{R}_1, \ldots, \mathcal{R}_n, \pi \rangle$$

- For local satisfiability, formulae are interpreted with respect to the root of $\mathcal{M}$, that is, w_0. A formula φ is *locally satisfied in* $\mathcal{M}$, denoted by $\mathcal{M} \models_L \varphi$, if $\langle \mathcal{M}, w_0 \rangle \models \varphi$.
- The formula φ is locally satisfiable if there is a model $\mathcal{M}$ such that $\langle \mathcal{M}, w_0 \rangle \models \varphi$.
- A formula φ is globally satisfied in $\mathcal{M}$, if for all $w \in \mathcal{W}$, $\langle \mathcal{M}, w \rangle \models \varphi$.
- A formula φ is globally satisfiable if there is a model $\mathcal{M}$ such that $\mathcal{M}$ globally satisfies φ, denoted by $\mathcal{M} \models_G \varphi$.
- Given a set of formulae Γ and a formula φ, the local satisfiability of φ under the global constraints Γ consists of showing that there is a model that globally satisfies the formulae in Γ and that there is a world in this model that satisfies φ.

Complexity

- Local satisfiability: PSPACE-complete;
- Global satisfiability: EXPTIME-complete;
- Local satisfiability under global constraints: EXPTIME-complete.

Proof Methods

- Translation into first-order logic;
- Sequent calculus;
- Tableaux;
- Inverse method;
- BDD;
- SAT;
- Resolution;
- . . .

Implementation

```
$./prover -i benchmarks/lwb/k_branch_p.01.ksp -fsub -ires
Unsatisfiable.
0.02 seconds
```

Implementation

```
$./prover -i benchmarks/lwb/k_branch_p.01.ksp -fsub -ires
Unsatisfiable.
0.02 seconds

$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires
^C
363.98 seconds
```

Implementation

```
$./prover -i benchmarks/lwb/k_branch_p.01.ksp -fsub -ires
Unsatisfiable.
0.02 seconds
```

```
$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires
^C
363.98 seconds
```

```
$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires -bnfsimp -bsub -unit -ple
Unsatisfiable.
0.14 seconds
```

Implementation

```
$./prover -i benchmarks/lwb/k_branch_p.01.ksp -fsub -ires
Unsatisfiable.
0.02 seconds

$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires
^C
363.98 seconds

$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires -bnfsimp -bsub -unit -ple
Unsatisfiable.
0.14 seconds

$./prover -i benchmarks/lwb/k_branch_p.03.ksp -fsub -ires -bnfsimp -bsub -unit -ple
Unsatisfiable.
0.49 seconds
```

Implementation

```
$./prover -i benchmarks/lwb/k_branch_p.01.ksp -fsub -ires
Unsatisfiable.
0.02 seconds

$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires
^C
363.98 seconds

$./prover -i benchmarks/lwb/k_branch_p.02.ksp -fsub -ires -bnfsimp -bsub -unit -ple
Unsatisfiable.
0.14 seconds

$./prover -i benchmarks/lwb/k_branch_p.03.ksp -fsub -ires -bnfsimp -bsub -unit -ple
Unsatisfiable.
0.49 seconds

$./prover -i benchmarks/lwb/k_branch_p.04.ksp -fsub -ires -bnfsimp -bsub -unit -ple
^C
118.26 seconds
```

Example

$$\Diamond\Diamond p \wedge \Box\neg p$$

1. $\textsf{start} \rightarrow t_0$
2. $t_0 \rightarrow \Diamond t_1$
3. $t_1 \rightarrow \Diamond p$
4. $t_0 \rightarrow \Box\neg p$

Previous work

- Areces, C., Gennari, R., Heguiabehere, J., de Rijke, M.: Tree-based heuristics in modal theorem proving. In: Proc. of ECAI 2000. pp. 199-203. IOS Press (2000).

$$\Diamond\Diamond p \wedge \Box\neg p \Longrightarrow \Diamond\Diamond p_2 \wedge \Box\neg p_1$$

Previous work

- Areces, C., Gennari, R., Heguiabehere, J., de Rijke, M.: Tree-based heuristics in modal theorem proving. In: Proc. of ECAI 2000. pp. 199-203. IOS Press (2000).

$$\Diamond\Diamond p \wedge \Box\neg p \Longrightarrow \Diamond\Diamond p_2 \wedge \Box\neg p_1$$

$$p \wedge \Box\neg p \Longrightarrow p_0 \wedge \Box\neg p_1$$

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- Areces, C., Gennari, R., Heguiabehere, J., de Rijke, M.: Tree-based heuristics in modal theorem proving. In: Proc. of ECAI 2000. pp. 199-203. IOS Press (2000).

$$\Diamond\Diamond p \wedge \Box\neg p \Longrightarrow \Diamond\Diamond p_2 \wedge \Box\neg p_1$$

$$p \wedge \Box\neg p \Longrightarrow p_0 \wedge \Box\neg p_1$$

- Areces, C., de Nivelle, H., de Rijke, M.: Prefixed Resolution: A Resolution Method for Modal and Description Logics. In: Ganzinger, H. (ed.) Proc. CADE-16. LNAI, vol. 1632, pp. 187-201. Springer, Berlin (Jul 7-10 1999).
  - Formulae labelled by either constants or pair of constants.
  - The inference rule for $\Diamond$ generates new labels.
  - The inference rule for $\Box$ corresponds to propagation.

The main idea

- The calculus should allow for both local and modal reasoning.
- A formula to be tested for (un)satisfiability is translated into a normal form, where labels refer to the modal level they occur.
- Inference rules are then applied by modal level.

The Normal Form

After translation we have formulae of the form:

$$ml : \varphi$$

where $ml \in \mathbb{N}$, denoting that φ holds at the modal level ml; or

$$* : \varphi$$

which denotes that φ holds everywhere in the model. That is, satisfiability of labelled formulae is given by:

- $\mathcal{M}^* \models_L ml : \varphi$ if, and only if, for all worlds $w \in \mathcal{W}$ such that depth$(w) = ml$, we have $\langle \mathcal{M}^*, w \rangle \models_L \varphi$;
- $\mathcal{M}^* \models_L * : \varphi$ if, and only if, $\mathcal{M}^* \models_L \boxed{*}\, \varphi$.

Clauses

- Literal clause $ml : \bigvee_{b=1}^{r} l_b$
- Positive a-clause $ml : l' \to \boxed{a} l$
- Negative a-clause $ml : l' \to \Diamond_{a} l$

where $ml \in \mathbb{N} \cup \{*\}$ and l, l', $l_b \in \mathcal{L}$. Positive and negative a-clauses are together known as *modal a-clauses*; the index a may be omitted if it is clear from the context.

Transformation Rules

$$
\begin{aligned}
\rho(ml : t \rightarrow \varphi \wedge \varphi') &= \rho(ml : t \rightarrow \varphi) \wedge \rho(ml : t \rightarrow \varphi') \\
\rho(ml : t \rightarrow \boxed{a}\varphi) &= (ml : t \rightarrow \boxed{a}\varphi), \text{ if } \varphi \text{ is a literal} \\
&= (ml : t \rightarrow \boxed{a}t') \wedge \rho(ml + 1 : t' \rightarrow \varphi), \text{ otherwise} \\
\rho(ml : t \rightarrow \Diamond_a\varphi) &= (ml : t \rightarrow \Diamond_a\varphi), \text{ if } \varphi \text{ is a literal} \\
&= (ml : t \rightarrow \Diamond_a t') \wedge \rho(ml + 1 : t' \rightarrow \varphi), \text{ otherwise} \\
\rho(ml : t \rightarrow \varphi \vee \varphi') &= (ml : \neg t \vee \varphi \vee \varphi'), \text{ if } \varphi' \text{ is a disjunction of literals} \\
&= \rho(ml : t \rightarrow \varphi \vee t') \wedge \rho(ml : t' \rightarrow \varphi'), \text{ otherwise}
\end{aligned}
$$

Inference Rules

[LRES]

$$\begin{array}{rccc} ml: & D & \vee & l \\ ml': & D' & \vee & \neg l \\ \hline \sigma(\{ml, ml'\}): & D & \vee & D' \end{array}$$

[MRES]

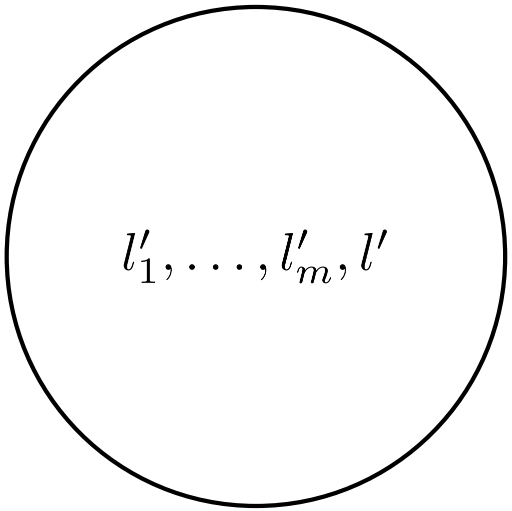
$$\begin{array}{rccc} ml: & l_1 & \rightarrow & \boxed{a} l \\ ml': & l_2 & \rightarrow & \Diamond_a \neg l \\ \hline \sigma(\{ml, ml'\}): & \neg l_1 & \vee & \neg l_2 \end{array}$$

Inference Rules

[GEN1]

$$\begin{array}{rl}
ml_1 : & l'_1 \rightarrow \boxed{a}\neg l_1 \\
& \vdots \\
ml_m : & l'_m \rightarrow \boxed{a}\neg l_m \\
ml_{m+1} : & l' \rightarrow \diamondsuit_a \neg l \\
ml_{m+2} : & l_1 \vee \ldots \vee l_m \vee l \\
\hline
ml : & \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'
\end{array}$$

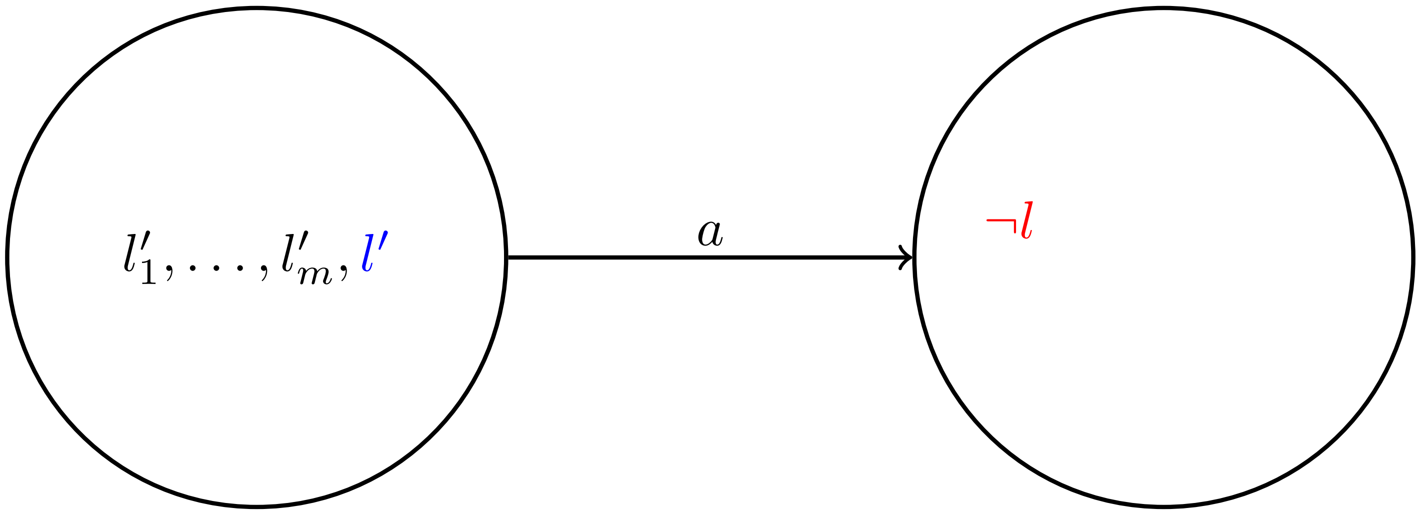
where $ml = \sigma(\{ml_1, \ldots, ml_{m+1}, ml_{m+2} - 1\})$



[GEN1]

$$
\begin{array}{rl}
ml_1 : & l'_1 \rightarrow \boxed{a}\neg l_1 \\
& \vdots \\
ml_m : & l'_m \rightarrow \boxed{a}\neg l_m \\
ml_{m+1} : & l' \rightarrow \Diamond_a \neg l \\
ml_{m+2} : & l_1 \vee \ldots \vee l_m \vee l \\
\hline
ml : & \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'
\end{array}
$$

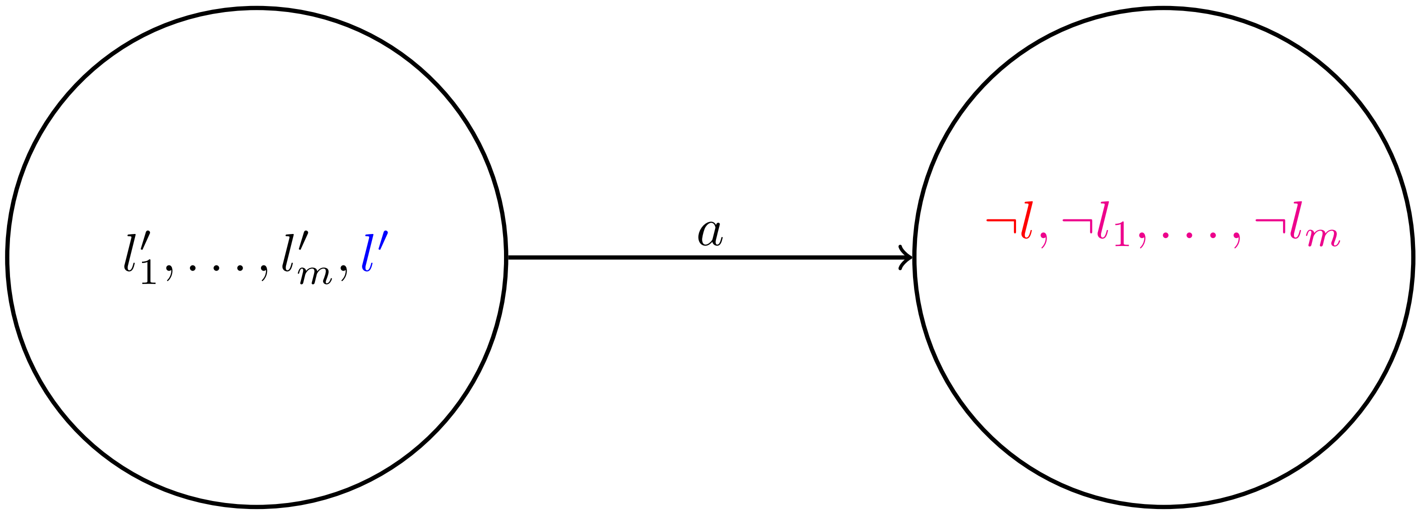
where $ml = \sigma(\{ml_1, \ldots, ml_{m+1}, ml_{m+2} - 1\})$



[GEN1]

$$
\begin{array}{rlcl}
ml_1: & l'_1 & \to & \boxed{a}\neg l_1 \\
 & \vdots & & \\
ml_m: & l'_m & \to & \boxed{a}\neg l_m \\
ml_{m+1}: & l' & \to & \Diamond_a \neg l \\
ml_{m+2}: & l_1 \vee \ldots \vee l_m \vee l & & \\
\hline
ml: & \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l' & &
\end{array}
$$

where $ml = \sigma(\{ml_1, \ldots, ml_{m+1}, ml_{m+2} - 1\})$

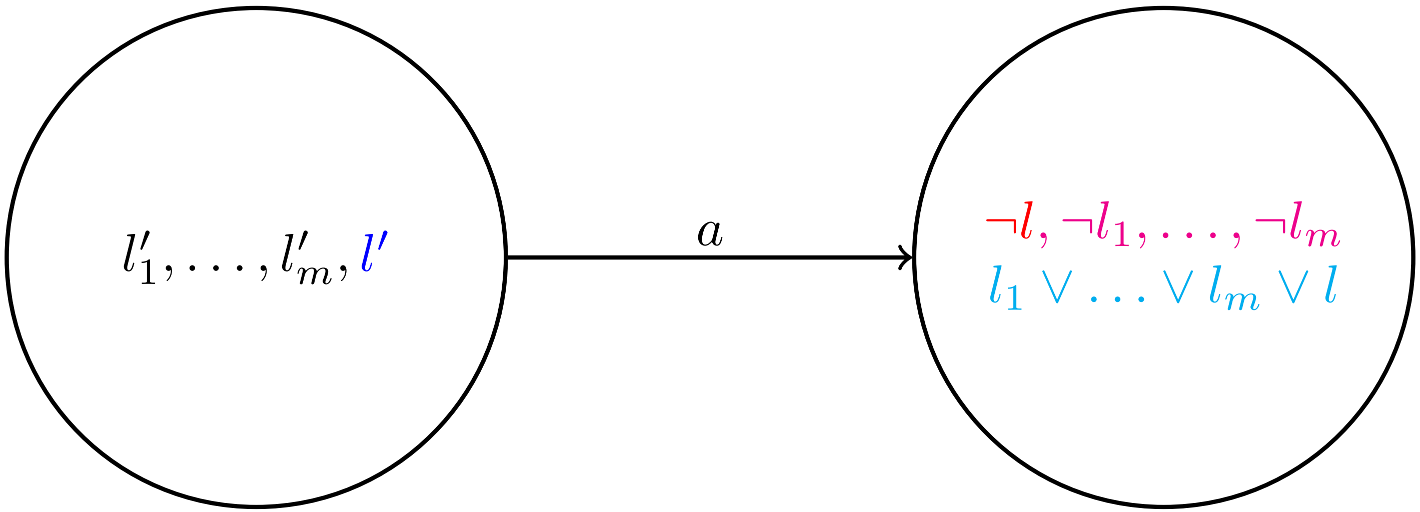


Inference Rules

[GEN1]

$$
\begin{array}{rcccl}
ml_1 : & l'_1 & \rightarrow & \boxed{a}\neg l_1 \\
 & \vdots & & \\
ml_m : & l'_m & \rightarrow & \boxed{a}\neg l_m \\
ml_{m+1} : & l' & \rightarrow & \Diamond_a \neg l \\
ml_{m+2} : & \multicolumn{3}{l}{l_1 \vee \ldots \vee l_m \vee l} \\
\hline
ml : & \multicolumn{3}{l}{\neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'}
\end{array}
$$

where $ml = \sigma(\{ml_1, \ldots, ml_{m+1}, ml_{m+2} - 1\})$



Inference Rules

[GEN2]

$$\frac{\begin{array}{rccc} ml_1 : & l'_1 & \rightarrow & \boxed{a} l_1 \\ ml_2 : & l'_2 & \rightarrow & \boxed{a} \neg l_1 \\ ml_3 : & l'_3 & \rightarrow & \Diamond_a l_2 \end{array}}{\sigma(\{ml_1, ml_2, ml_3\}) : \quad \neg l'_1 \vee \neg l'_2 \vee \neg l'_3}$$

Inference Rules

[GEN3]

$$\begin{array}{rl} ml_1: & l'_1 \rightarrow \boxed{a}\neg l_1 \\ & \vdots \\ ml_m: & l'_m \rightarrow \boxed{a}\neg l_m \\ ml_{m+1}: & l' \rightarrow \Diamond_a l \\ ml_{m+2}: & l_1 \vee \ldots \vee l_m \\ \hline ml: & \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l' \end{array}$$

where $ml = \sigma(\{ml_1, \ldots, ml_{m+1}, ml_{m+2} - 1\})$

Example

1.	$* :$	$female \vee male$
2.	$* :$	$\neg female \vee \neg male$
3.	$* :$	$\neg tall \vee t_1$
4.	$* :$	$t_1 \rightarrow \boxed{c} blond$
5.	$0 :$	t_0
6.	$0 :$	$t_0 \rightarrow \boxed{c} t_2$
7.	$1 :$	$\neg t_2 \vee \neg female \vee tall$
8.	$0 :$	$t_0 \rightarrow \langle c \rangle t_3$
9.	$1 :$	$t_3 \rightarrow \langle c \rangle \neg blond$
10.	$0 :$	$t_0 \rightarrow \boxed{c} \neg male$

Example

1.	$*:$	$female \vee male$	
2.	$*:$	$\neg female \vee \neg male$	
3.	$*:$	$\neg tall \vee t_1$	
4.	$*:$	$t_1 \rightarrow \boxed{c} blond$	
5.	$0:$	t_0	
6.	$0:$	$t_0 \rightarrow \boxed{c} t_2$	
7.	$1:$	$\neg t_2 \vee \neg female \vee tall$	
8.	$0:$	$t_0 \rightarrow \Diamond_c t_3$	
9.	$1:$	$t_3 \rightarrow \Diamond_c \neg blond$	
10.	$0:$	$t_0 \rightarrow \boxed{c} \neg male$	
11.	$1:$	$\neg t_1 \vee \neg t_3$	$[\mathsf{MRES}, 9, 4, blond]$

Example

1.	$*:$	$female \lor male$	
2.	$*:$	$\neg female \lor \neg male$	
3.	$*:$	$\neg tall \lor t_1$	
4.	$*:$	$t_1 \rightarrow \boxed{c}\, blond$	
5.	$0:$	t_0	
6.	$0:$	$t_0 \rightarrow \boxed{c}\, t_2$	
7.	$1:$	$\neg t_2 \lor \neg female \lor tall$	
8.	$0:$	$t_0 \rightarrow \Diamond_c\, t_3$	
9.	$1:$	$t_3 \rightarrow \Diamond_c\, \neg blond$	
10.	$0:$	$t_0 \rightarrow \boxed{c}\, \neg male$	
11.	$1:$	$\neg t_1 \lor \neg t_3$	$[\mathsf{MRES}, 9, 4, blond]$
12.	$1:$	$\neg tall \lor \neg t_3$	$[\mathsf{LRES}, 11, 3, t_1]$

Example

1.	$*:$	$\mathit{female} \vee \mathit{male}$	
2.	$*:$	$\neg \mathit{female} \vee \neg \mathit{male}$	
3.	$*:$	$\neg \mathit{tall} \vee t_1$	
4.	$*:$	$t_1 \rightarrow \boxed{c}\, \mathit{blond}$	
5.	$0:$	t_0	
6.	$0:$	$t_0 \rightarrow \boxed{c}\, t_2$	
7.	$1:$	$\neg t_2 \vee \neg \mathit{female} \vee \mathit{tall}$	
8.	$0:$	$t_0 \rightarrow \Diamond_c t_3$	
9.	$1:$	$t_3 \rightarrow \Diamond_c \neg \mathit{blond}$	
10.	$0:$	$t_0 \rightarrow \boxed{c}\, \neg \mathit{male}$	
11.	$1:$	$\neg t_1 \vee \neg t_3$	$[\mathsf{MRES}, 9, 4, \mathit{blond}]$
12.	$1:$	$\neg \mathit{tall} \vee \neg t_3$	$[\mathsf{LRES}, 11, 3, t_1]$
13.	$1:$	$\neg t_3 \vee \neg t_2 \vee \neg \mathit{female}$	$[\mathsf{LRES}, 7, 12, \mathit{tall}]$

Example

1.	$* :$	$female \vee male$	
2.	$* :$	$\neg female \vee \neg male$	
3.	$* :$	$\neg tall \vee t_1$	
4.	$* :$	$t_1 \rightarrow \boxed{c}\, blond$	
5.	$0 :$	t_0	
6.	$0 :$	$t_0 \rightarrow \boxed{c}\, t_2$	
7.	$1 :$	$\neg t_2 \vee \neg female \vee tall$	
8.	$0 :$	$t_0 \rightarrow \Diamond_{c}\, t_3$	
9.	$1 :$	$t_3 \rightarrow \Diamond_{c} \neg blond$	
10.	$0 :$	$t_0 \rightarrow \boxed{c} \neg male$	
11.	$1 :$	$\neg t_1 \vee \neg t_3$	$[\mathsf{MRES}, 9, 4, blond]$
12.	$1 :$	$\neg tall \vee \neg t_3$	$[\mathsf{LRES}, 11, 3, t_1]$
13.	$1 :$	$\neg t_3 \vee \neg t_2 \vee \neg female$	$[\mathsf{LRES}, 7, 12, tall]$
14.	$1 :$	$male \vee \neg t_2 \vee \neg t_3$	$[\mathsf{LRES}, 13, 1, tall]$

Example

1.	$* :$	$\mathit{female} \lor \mathit{male}$	
2.	$* :$	$\neg \mathit{female} \lor \neg \mathit{male}$	
3.	$* :$	$\neg \mathit{tall} \lor t_1$	
4.	$* :$	$t_1 \rightarrow \boxed{c}\, \mathit{blond}$	
5.	$0 :$	t_0	
6.	$0 :$	$t_0 \rightarrow \boxed{c}\, t_2$	
7.	$1 :$	$\neg t_2 \lor \neg \mathit{female} \lor \mathit{tall}$	
8.	$0 :$	$t_0 \rightarrow \langle c \rangle t_3$	
9.	$1 :$	$t_3 \rightarrow \langle c \rangle \neg \mathit{blond}$	
10.	$0 :$	$t_0 \rightarrow \boxed{c}\, \neg \mathit{male}$	
11.	$1 :$	$\neg t_1 \lor \neg t_3$	$[\mathsf{MRES}, 9, 4, \mathit{blond}]$
12.	$1 :$	$\neg \mathit{tall} \lor \neg t_3$	$[\mathsf{LRES}, 11, 3, t_1]$
13.	$1 :$	$\neg t_3 \lor \neg t_2 \lor \neg \mathit{female}$	$[\mathsf{LRES}, 7, 12, \mathit{tall}]$
14.	$1 :$	$\mathit{male} \lor \neg t_2 \lor \neg t_3$	$[\mathsf{LRES}, 13, 1, \mathit{tall}]$
15.	$0 :$	$\neg t_0$	$[\mathsf{GEN1}, 10, 6, 8, 14, \mathit{male}, t_2, t_3]$

Example

1.	$* :$	$\mathit{female} \lor \mathit{male}$	
2.	$* :$	$\neg \mathit{female} \lor \neg \mathit{male}$	
3.	$* :$	$\neg \mathit{tall} \lor t_1$	
4.	$* :$	$t_1 \to \boxed{c}\, \mathit{blond}$	
5.	$0 :$	t_0	
6.	$0 :$	$t_0 \to \boxed{c}\, t_2$	
7.	$1 :$	$\neg t_2 \lor \neg \mathit{female} \lor \mathit{tall}$	
8.	$0 :$	$t_0 \to \langle c \rangle t_3$	
9.	$1 :$	$t_3 \to \langle c \rangle \neg \mathit{blond}$	
10.	$0 :$	$t_0 \to \boxed{c}\, \neg \mathit{male}$	
11.	$1 :$	$\neg t_1 \lor \neg t_3$	$[\mathsf{MRES}, 9, 4, \mathit{blond}]$
12.	$1 :$	$\neg \mathit{tall} \lor \neg t_3$	$[\mathsf{LRES}, 11, 3, t_1]$
13.	$1 :$	$\neg t_3 \lor \neg t_2 \lor \neg \mathit{female}$	$[\mathsf{LRES}, 7, 12, \mathit{tall}]$
14.	$1 :$	$\mathit{male} \lor \neg t_2 \lor \neg t_3$	$[\mathsf{LRES}, 13, 1, \mathit{tall}]$
15.	$0 :$	$\neg t_0$	$[\mathsf{GEN1}, 10, 6, 8, 14, \mathit{male}, t_2, t_3]$
16.	$0 :$	false	$[\mathsf{LRES}, 15, 5, t_0]$

Negative Resolution

- A literal is negative if is of the form $\neg p$, where $p \in \mathcal{P}$.
- A clause C is negative if all literals in C are negative.
- Negative resolution restricts the application of the inference rules by requiring that one of the clauses being resolved is negative.
- For completeness, we need to change the normal form:

$$\begin{array}{rcl} \rho(ml : t \to \boxed{a}\neg p) & = & (ml : t \to \boxed{a} t') \wedge \rho(ml+1 : t' \to \neg p) \\ \rho(ml : t \to \Diamond_{a}\neg p) & = & (ml : t \to \Diamond_{a} t') \wedge \rho(ml+1 : t' \to \neg p) \end{array}$$

Ordered Resolution

- Let Φ be a set of clauses and $\mathcal{P}_\Phi$ be the set of propositional symbols occurring in Φ.
- Let $\succ$ be a well-founded and total ordering on $\mathcal{P}_\Phi$.
- This ordering can be extended to literals $\mathcal{L}_\Phi$ occurring in Φ by setting $\neg p \succ p$ and $p \succ \neg q$ whenever $p \succ q$, for all $p, q \in \mathcal{P}_\Phi$.
- A literal l is said to be *maximal* with respect to a clause $C \vee l$ if, and only if, there is no l' occurring in C such that $l' \succ l$.
- Two clauses $C \vee l$ and $C' \vee \neg l$ can be resolved if, and only if, l is maximal with respect to C and $\neg l$ is maximal with respect to C'.
- For completeness, we have to make sure that every literal occurring in the scope of a modal operator is minimal with respect to the other literals occurring at the same modal level.
- For the running example (k_branch_p.04), negative resolution reports unsatisfiability in 4.14 seconds whilst ordered resolution takes 0.05 seconds.

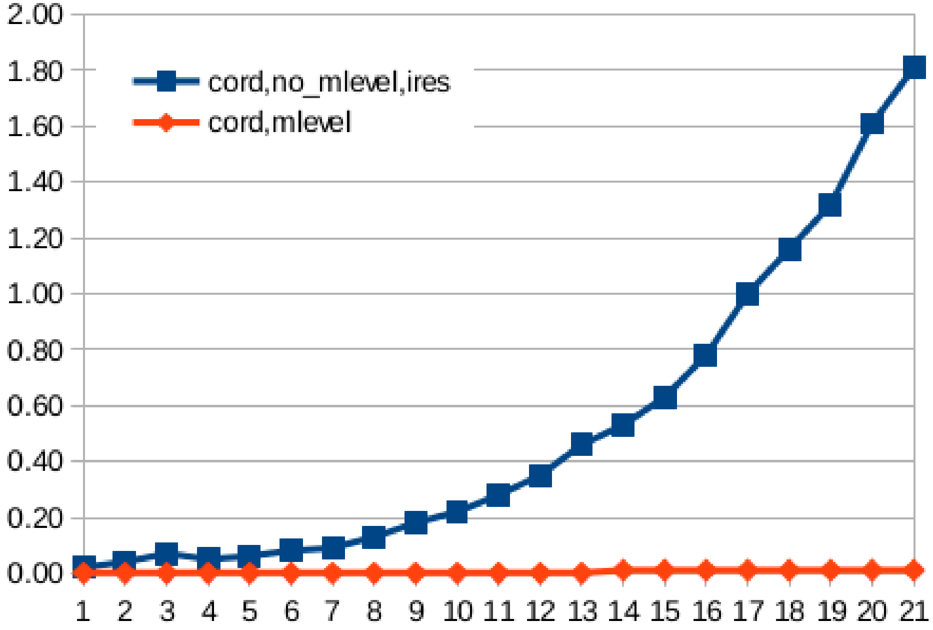


Figure 1: Unsatisfiable Formulae

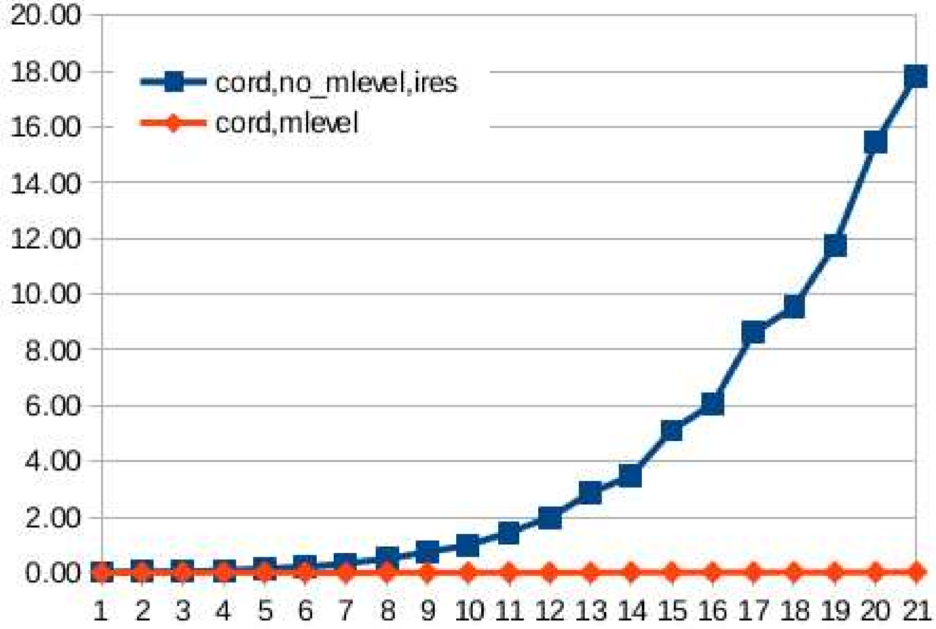
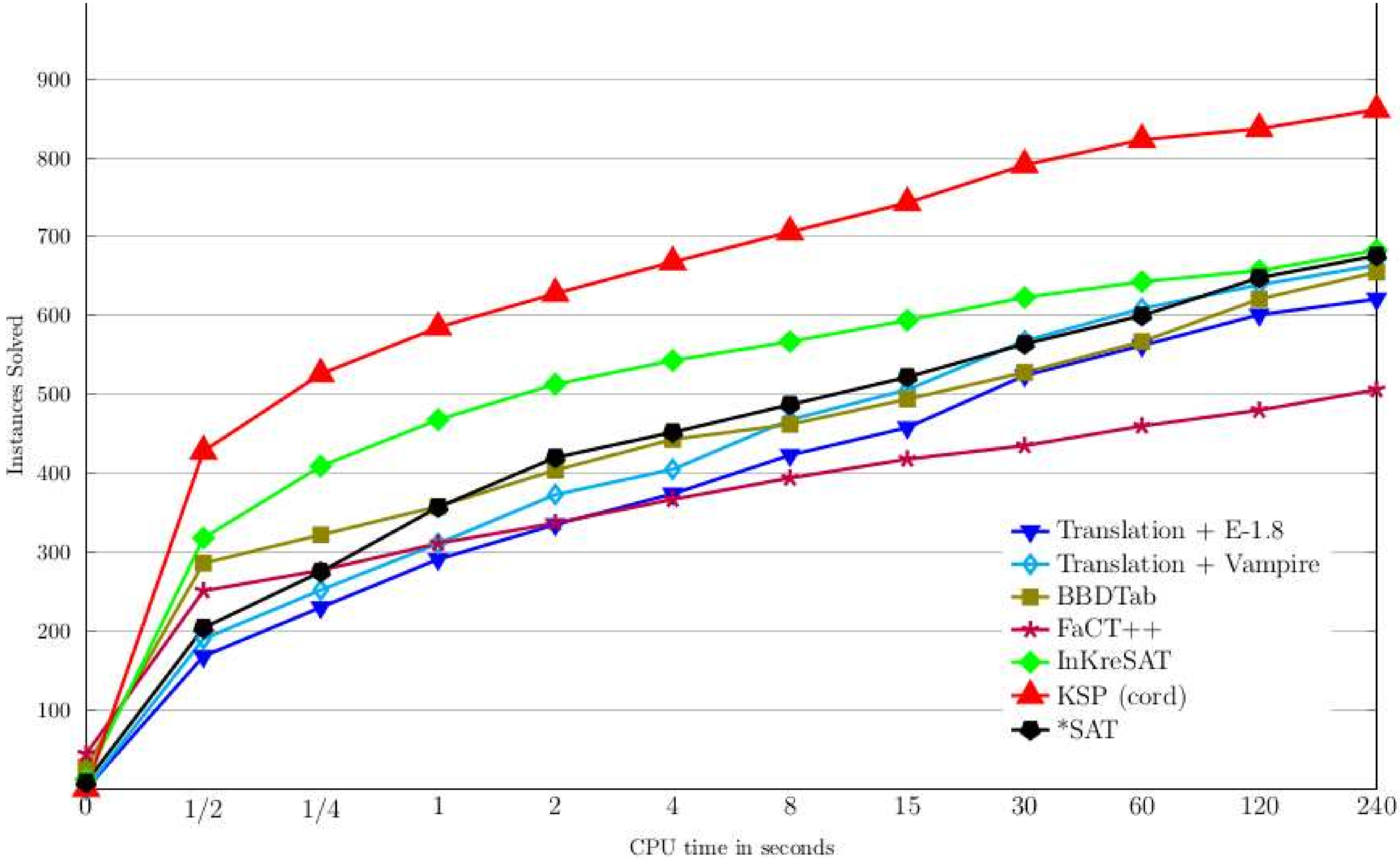


Figure 2: Satisfiable Formulae

QBF



900
800
700
600
500
400
300
200
100
Instances Solved
0
1/2
1/4
1
2
4
8
15
30
60
120
240
CPU time in seconds
Translation + E-1.8
Translation + Vampire
BBDTab
FaCT++
InKreSAT
KSP (cord)
*SAT

Conclusion and Future Work

- We have presented a terminating, sound, and complete (non-natural, polluted) calculus for K_n.
- Negative and ordered resolution, together with layering, are also complete.
- Implementation is still work in progress, but results seem to be promising.
- We are considering other refinements as negative ordered resolution, for instance.