On the Coexistence of Cognitive Radio and Cellular Networks: An Outage Analysis

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Abstract— **In this paper, coexistence of cellular network with cognitive radio (CR) users or secondary users (SUs) has been studied. More precisely, an analytical frame work for downlink interference modelling in a cognitive radio frequency planned environment has been developed. Outage performance of a PU in a frequency planned cognitive radio network has been investigated. Dynamic spectrum sharing with spectrum underlay approach has been considered. Further, we assume correlation amongst different log normal interferers from several co-channel BSs and cognitive users. A simulation test bed for evaluating outage probability in cognitive radio cellular networks has been developed. Effects of several network and channel parameters are shown on outage probability of a PU.**

Keywords- Outage probability, Cognitive radio, Cellular Network, Correlated interference.

I. INTRODUCTION

Cognitive radio is a promising candidate for future and present generation network as spectrum is getting scarce day by day. The term, cognitive radio, was first coined by Mitola in 1999 [1]. Cognitive radio networks allow presence of primary users (PUs) and secondary users (SUs). In a cognitive radio network, secondary user may change its radio parameters on demand. For example, it can adapt its rate when the numbers of primary users are smaller or the interference level is low [2]. Spectrum sensing relates to finding unused spectrum by the SU i.e., the spectrum that is presently not used by PUs or being used by SUs with interference level below interference temperature limit. After finding spectrum holes [3], i.e., unused spectrum in spatial and/or temporal domains, a SU selects the best available channel, and this is known as *spectrum decision*. Dynamic spectrum access (DSA) relates to spectrum usage by SUs whenever there is an opportunity. A SU can avail the best channel by DSA. Cognitive users can coexist with primary users in two ways, either through spectrum underlay or spectrum overlay [4].

On the other hand, cellular network has been very popular, so far, due to several reasons. Cellular network offers good quality of voice service with limited data service. But, main reason of its popularity is mobility of users. Users can talk while they move. Second generation and third generation wireless communication networks are mainly based on cellular network. Hence, coexistence study of cellular network with future generation cognitive radio network is very much useful. Moreover, coexistence study of cellular and cognitive radio network has already attracted attention of researchers [5].

In our present paper, we assume cognitive radio (CR) users or SUs in ad hoc mode using spectrum licensed to cellular operators in a frequency planned cellular structure as in [5]. We study the impact of secondary users and several other parameters (of channel and network scenarios) on the outage probability of a PU. We consider a downlink SIR threshold at the PU of interest. QoS at the PU terminal can not be maintained if instantaneous interference is below the SIR threshold. All PUs measure downlink interference and give the information to their base station. A BS periodically broadcasts information regarding increase in interference limit to all PUs and SUs, once it obtains the same from all users. On the basis of broadcast information, a SU may decrease its power or it may stop transmitting temporarily. To get an overall idea, on the number of cognitive users allowed in the system, operating zone, effects of shadowing on overall performance, we consider that PU would go to outage if SIR falls below a given threshold. More precisely, we provide a simulation test bed with reasonable analytical framework for evaluating total downlink interference at a PU due to co-channel BSs and other SUs. We consider shadow fading in our channel model. A number of shadow faded signals from several independent sources may be received at a user terminal. Those shadow faded signals may be correlated if they are obstructed by same obstacle near receiver [8]. Hence, it is expected that interference caused by co-channel BSs and SUs at a given PU would be correlated. Performance of cognitive radio in a frequency planned cellular environment has been investigated in [5] without considering such correlation amongst interferers.

We consider the effects of correlation amongst the interferers on the outage performance of a PU in such an environment. Our contributions in this paper are two fold. First, we have extended the model of [5] without correlation among interferers to a more realistic scenario with correlation among interferers and analyzed performance of the network for the extended model. Secondly, we have derived an expression for outage probability of a PU for such a model. Next in Section II, we present the basics of the system that is modelled. Section II also gives analytical details along with the calculation of SIR and probability of outage. In Section III, we describe our simulation model. In Section IV, selected numerical results are illustrated to substantiate the efficacy of the analysis. The paper concludes in Section V.

II. System Model

In our assumed model, each cell is covered by one BS using an omni directional antenna. In our analysis, both PU and SU use omni directional antennas. Primary users are distributed inside the inner circle of BS_0 as shown in Fig. 1. All base stations are distributed as shown in the Fig. 1. We derive outage probability expression for a primary user in presence of secondary users using the same spectrum in an underlay spectrum sharing architecture.

Received power at a given primary user, at distance d_i from any BS, assuming log-normal shadowing (neglecting multipath fading), can be expressed as follows:

$$P_{rx} = P_t.10^{\frac{\zeta_i}{10}}.d_i^{-n} \tag{1}$$

Let us assume that P_t is the transmitted power from any other *BS* in downlink. Received power (in dB) from any BS may be expressed as follows:

$$P_{rx}(dB) = K_1 - K_2.\log10(d_i) + \zeta_i \tag{2}$$

K_1 and K_2 depend on transmit power from base station and path loss exponent. Similarly, the received power at a PU from a cognitive radio user can be expressed (in dB) as follows:

$$P_{r\_cog}(dB) = K_{1,cog} - K_{2,cog}.\log10(d_{i,cog}) + \zeta_{i,cog} \tag{3}$$

where, $K_{1,cog}$ and $K_{2,cog}$ are constants based on transmit power from cognitive user and path loss exponent. Actually, the received power at a primary user with polar co-ordinate (r,θ) from BS_0 (with reference to Fig. 1) is,

$$P_{PU\_d} = \phi.P_t.10^{\frac{\zeta_i}{10}}.d_i^{-n} \tag{4}$$

where, $\phi.P_t$ = fractional power received from BS_0 i.e., available power from BS_0 subtracting pilot power /no. of primary users. A constant value of ϕ is assumed (= 1 in this case) without loss of generality. A PU may be present any where within the cell. Received power from BS_0 at a PU at a location (r,θ) of BS_0 is

$$P_r^0 = P_t.r^{-n}.10^{\frac{\xi}{10}} \tag{5}$$

Now we want to find the mean of received power from other co-channel base station, BS_i (i=1 to 6). The distance, d_i from a co-channel BS to a PU at a location (r,θ) of BS_0 can be easily found considering the geometry of Fig. 1. Overall downlink interference from six co-channel BSs at the PU of interest is found as follows:

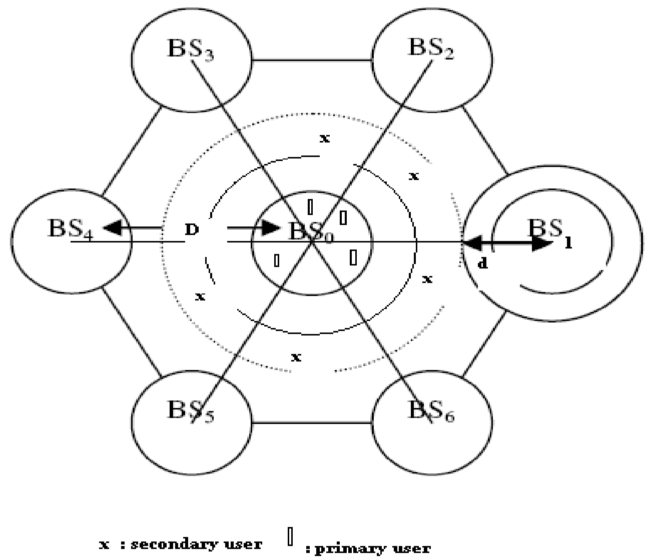

Fig. 1. Hexagonal multi-cellular model with seven BSs labelled as 0, 1, 2, 3 ….6. PUs are inside the inner circle with radius r_0 around BS_0. BSs are separated from one another by *D* meter. SUs are inside the ring within two circles with radii of d and $(D-d)$ respectively, where $r_0 < d < (D-d)$.

$$I_1 = \sum_{i=1}^{6} P_t.10^{\frac{\zeta_i}{10}}.d_i^{-n} \tag{6}$$

where, d_i is distance from *i*-th co-channel BS and ζ_i is due to shadowing in the channel. $\zeta_i(dB)$ has zero mean and variance, σ^2. Next, we find the interference caused by CR users to the PU under consideration. Let us assume that N_{cog} is the total number of secondary users or cognitive users present in the system. Further, we assume that they are inside the annular ring; i.e., within the distance, *d* to (*D*-*d*) as in Fig. 1. P_{cog} is the power transmitted by any cognitive user. We assume that, $r_0 \le d \le \frac{D}{2}$ and we consider d_m to be the distance from any cognitive user to the desired primary user which is given as:

$$d_m = \sqrt{(x_m - x)^2 + (y_m - y)^2} \tag{7}$$

where x_m , y_m are co-ordinates of CR user in their specified zone and x , y are co-ordinates of desired primary user (in rectangular co-ordinate).

$$d_m = \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2} \tag{8}$$

Here, r_2, θ_2 are polar co-ordinates indicating position of a CR user. r_2 is between d and $(D-d)$. θ_2 is uniformly distributed between $(0, 2\pi)$. Similarly, r_1, θ_1 indicates position of a primary user in polar co-ordinate. The distance r_1 is assumed to be within $(0, r_0)$. Overall interference at the PU of interest from all these SUs may be expressed as:

$$I_2 = \sum_{m=1}^{N_{cog}} P_{cog,m}.10^{\frac{\zeta_{cog,m}}{10}}.d_m^{-n} \quad (9)$$

where, d_m is the distance of the PU from m-th SU and $\zeta_{cog,m}$ is due to shadowing in the channel. $P_{cog,m}$ is the transmit power of m-th cognitive user.

Now, we attempt to find the *SINR* expression at the desired primary user as:

$$SINR = \frac{P_t.10^{\frac{\zeta_i}{10}}.d_i^{-n}}{\sum_{j=1}^{6} P_t.10^{\frac{\zeta_j}{10}}.d_j^{-n} + \sum_{m=1}^{N_{cog}} P_{cog,m}.10^{\frac{\zeta_{cog,m}}{10}}.d_m^{-n} + N} \quad (10)$$

where N is the variance of noise power. We have assumed an interference dominated situation, and we have neglected the noise term in our analysis. The above *SINR* expression becomes *SIR* expression as follows:

$$SIR = \frac{P_t.10^{\frac{\zeta_i}{10}}.d_i^{-n}}{\sum_{j=1}^{6} P_t.10^{\frac{\zeta_j}{10}}.d_j^{-n} + \sum_{m=1}^{N_{cog}} P_{cog,m}.10^{\frac{\zeta_{cog,m}}{10}}.d_m^{-n}} \quad (11)$$

The above *SIR* expression may also be written as,

$$SIR = \frac{e^{y_0}}{\sum_{i=1}^{6} e^{y_i} + \sum_{i=1}^{N_{cog}} e^{z_i}} = \frac{e^{y_0}}{e^X} \quad (12)$$

where, $y_i = \lambda P_{rx,i}(dB); \quad \lambda = 0.1 \ln 10$ (13)

y_i and y_0 are random variables with mean and variances as follows,

$m_{y0} = \lambda P_r^0(dB)$, $\sigma_{y_i}^2 = \lambda^2 \sigma_{P_r^0}^2$ and

$$m_{y_i} = E(\lambda P_{rx,i}(dB)), \quad \sigma_{y0}^2 = \lambda^2 \sigma_{rx,i}^2 \quad (14)$$

In the same way, we can obtain mean and variance of interference caused by SU as follows:

$$m_{z_i} = \lambda P_{r\_cog,i}(dB) \quad \text{and} \quad \sigma_{z_i}^2 = \lambda^2 \sigma_{r\_cog,i}^2 \quad (15)$$

Hence, mean and variance of *SIR* may be obtained from previous *SIR* expression (17).

$$m_{SIR} = m_{y0} - m_X \quad \text{and} \quad \sigma_{SIR}^2 = \sigma_{y0}^2 + \sigma_X^2 \quad (16)$$

Denominator of *SIR* expression as in (17),

$$L_n = e^X = \sum_{j=1}^{6} e^{y_i} + \sum_{m=1}^{N_{cog}} e^{z_i} \quad (17)$$

where, L_n is a log-normal random variable with mean and variance as given below.

$$E[L_n] = E[e^X] = E\left[\sum_{\forall i} e^{y_i}\right] + E\left[\sum_{\forall i} e^{z_i}\right] \quad (18)$$

We consider two variables u_1 and u_2 as given below.

$$u_1 = e^{m_X + \sigma_X^2} = \sum_{i=1}^{6} e^{m_{y_i} + \sigma_{y_i}^2} + \sum_{i=1}^{N_{cog}} e^{m_{z_i} + \sigma_{z_i}^2} \quad (19)$$

$$E[e^{2X}] = E[(\sum e^{y_i})^2] + E[(\sum e^{z_i})^2] + 2.E[\sum e^{y_i}].E[\sum e^{z_i}] \quad (20)$$

$$u_2 = e^{2m_X + 2\sigma_X^2} = E[(\sum e^{y_i})^2] + E[(\sum e^{z_i})^2] + 2.E[\sum e^{y_i}].E[\sum e^{z_i}] \quad (21)$$

We assume that r_{ij} is the pair wise correlation factor between a pair of lognormal interferers, i and j. Here lognormal interferers are due to downlink interference from co-channel BSs and due to interference from other SUs. Correlation between any pair of interferers is assumed to be identical for simplicity i.e., $r_{ij} = r$ for all *i, j*. Now, we can calculate mean and variance of signal to interference ratio, *SIR* as we know all the variances and means [8]:

$$m_X = 2 \ln u_1 - \frac{1}{2} \ln u_2 \quad \text{and} \quad \sigma_X^2 = \ln u_2 - 2 \ln u_1 \quad (22)$$

The probability of outage for a desired primary user is given as:

$$P_{out} = \Pr ob.(SIR \le SIR_{thd}) = 1 - Q\left(\frac{\log_e(SIR_{thd}) - m_{SIR}}{\sigma_{SIR}}\right) \quad (23)$$

We consider P_{out} as performance metric for our analysis. We have found the average downlink interference caused by co-channel BSs and CR users on PU using simulation described in the following section.

III. SIMULATION MODEL

The simulation is developed in MATLAB. In our simulation, parameters mentioned in Section IV are used.

A. Generation of Users' Locations

1. A PU is generated within BS_0 with coordinate (r, θ). This user is generated considering uniform distribution of users within the cell.

2. Considering coordinate of the centre of BS_0 as $(0,0)$, coordinate of all other co-channel BSs are found. For example coordinate of BS_1 is $(D,0)$. Similarly, coordinates

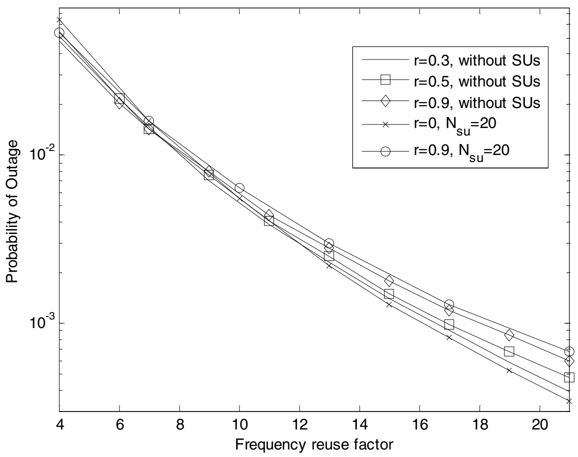

Fig. 2. Probability of outage for a PU vs. frequency reuse factor. Effects of correlation are shown.

of all other BSs are $\left(\frac{D}{2},\frac{\sqrt{3}}{2}D\right)$, $\left(-\frac{D}{2},\frac{\sqrt{3}}{2}D\right)$, $\left(-D,0\right),\left(-\frac{D}{2},-\frac{\sqrt{3}}{2}D\right),\left(\frac{D}{2},-\frac{\sqrt{3}}{2}D\right)$, respectively.

3. Now, distances from these co-channel BSs to the desired PU are calculated. Link gains from corresponding BSs and received powers at the PU of interest, are evaluated in each iteration.

4. A number of SUs $\left(N_{cog}\right)$are generated within the annular ring i.e., within the distance, *d* to (*D-d*) as shown in Fig. 1. Cognitive radio users are generated considering uniform distribution within the annular ring (as shown in Fig. 1).

5. The received power from each SU at the PU of interest is evaluated as discussed in Section 2.

6. Finally, SIR of the PU of interest is found.

B. Probability of Outage for PU

The following steps are followed.

1. The SIR is generated for a desired user as shown in the previous section and compared with a threshold value given by SIR_{th} .

2. If the SIR falls below SIR_{th} , an outage counter ($outage_{count}$) is incremented.

3. Steps (2) and (3) are repeated a large ($N_t >> 1$) number of times to yield an accurate estimate of the probability of outage as $P_{out} = outage_{count} / N_t$.

IV. RESULTS AND DISCUSSIONS

Following values of parameters are assumed to generate results based on analytical formulations and corresponding simulations described in Section II and III, respectively. The standard deviation of shadow fading (sig) is σ = 6, 8 dB, the distance between BSs is $D = r_0\sqrt{3q}$ meter, the radius of primary users zone is $r_0 = 500$ meter. Here, q is frequency reuse factor or cluster size. We consider frequency reuse factors of 7 and 21. We also assume that P_t = 1 W and P_{cog} = 1 mW with path loss exponent, $n = 4$. The number of secondary users present in the system is considered to be in the range of 5 to 25. Correlation coefficient is assumed to be 0, 0.5 and 0.9.

In Fig. 2, the probability of outage for a PU is shown as a function of frequency reuse factor of the cellular network. Effects of correlation amongst interferers from co-channel BSs and SUs on PU's performance have been indicated. The curves without SUs in this figure replicates results of Abu Daya's work [8]. Probability of outage decreases with increase in frequency reuse factor of the cellular network when frequency reuse factor is sufficiently high (>12). However, it can be observed that probability of outage decreases with decrease in correlation coefficient when frequency reuse factor is low. There is a clear cross over of plots at some values of outage probabilities. Probability of outage is increasing with increase in number of CR users.

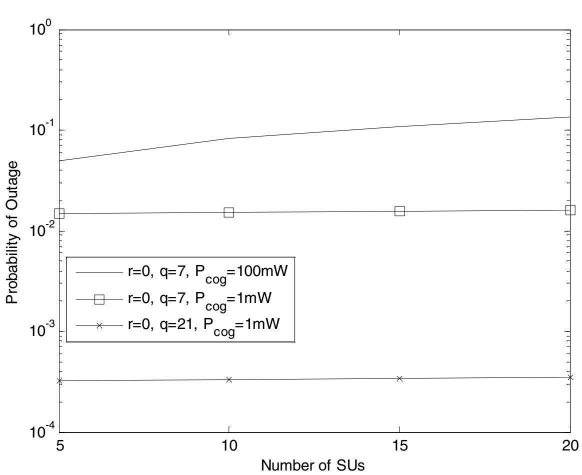

Fig. 3. Probability of outage for PU vs. the number of secondary users. Effects of SUs' power level and frequency reuse factor are shown.

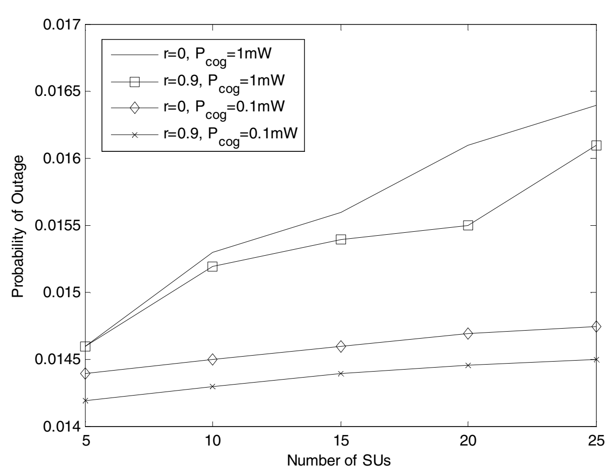

Fig. 4. Probability of outage for a PU vs. the number of secondary users for different values of P_{cog} = 0.1 mW and 1mW.

Higher values of correlation result in higher outage probability. As interferences from co-channel BSs and SUs get correlated, outage of PU increases.

Effects of frequency reuse factor of cellular network and transmit power from CR user on PU are shown in Fig. 3. Effects of two different levels of cognitive user's power and effects of two different frequency reuse factors on PU are depicted. As P_{cog} increases from 1 mW to 100 mW, probability of outage for PU increases to a large extent. This is due to increase in interference caused by SUs. However, probability of outage decreases heavily when frequency reuse factor is increased from 7 to 21. This is due to increase in distances of the PU of interest from co-channel BSs and corresponding reduction in downlink interference.

In Fig. 4, the probability of outage for a PU is shown as a function of number of secondary users. Effects of two different levels of CR user's power and effects of two different values of correlation co-efficient on PU are depicted. Increase in correlation among interferers increases probability of outage of the PU of interest. This is due to higher values of interference from co-channel BS and SUs. Thus, higher degree of correlation among interferences decreases overall interference. We consider frequency reuse factor of seven (7). Probability of outage decreases if transmit power of a SU is reduced.

In Fig. 5, the probability of outage for a PU is shown as a function of number of secondary users for different values of standard deviation of shadowing i.e., σ = 6 and 8 dB. Probability of outage increases if standard deviation of shadowing increases from 6 to 8 dB. Probability of outage for a PU decreases with decrease in correlation coefficient. We consider three different values of correlation coefficient (r = 0, 0.5. 0.9) for this figure. The probability of outage is lowest when r = 0.9. It increases if r is decreased to 0.5 from r = 0.9. Finally, it is highest for correlation coefficient of 0. As the standard deviation of shadow fading increases from 6 to 8 dB, high level of interference is created at PU. We see that the probability of outage for a PU increases with decrease in correlation among interferers.

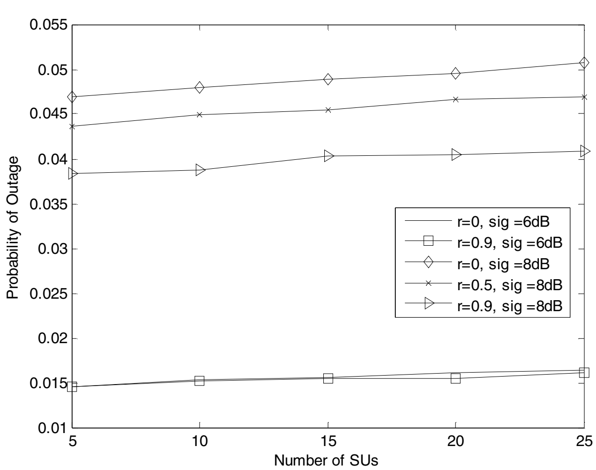

Fig. 5. Probability of outage for PU vs. number of secondary users. Comparison for probability of outage w.r.t number of secondary users is shown for σ = 6 and 8 dB.

V. CONCLUSION

This paper analyzes primary user's performance in a frequency planned cellular network in presence of cognitive radio users considering correlation amongst interferers from co-channel BSs, and correlation amongst interferers from SUs. Primary user's performance would improve if correlation amongst interferers decreases. Transmit power of CR users has significant impact on system performance. PU's performance degrades if transmit power from cognitive user is increased. Performance of a PU would improve if frequency reuse factor is increased. Our simulation model is helpful for performance evaluation of a frequency planned network consisting of PU and SU. This simulation model would be helpful for network designers who want to integrate 2G and 3G cellular network with future generation wireless network that employs cognitive radio.

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