Uplink Soft Frequency Reuse for Self-Coexistence of Cognitive Radio Networks Operating in White-Space Spectrum

Bo Gao, Jung-Min "Jerry" Park, and Yaling Yang
Bradley Department of Electrical and Computer Engineering
Virginia Tech, Blacksburg, VA 24061, USA
Email: {bgao, jungmin, yyang8}@vt.edu

***Abstract*—Recent advances in cognitive radio (CR) technology have brought about a number of wireless standards that support opportunistic access to available white-space spectrum. Addressing the self-coexistence of CR networks in such an environment is very challenging, especially when coexisting networks operate in the same swath of spectrum with little or no direct coordination. In this paper, we study the problem of co-channel self-coexistence of uncoordinated CR networks that employ orthogonal frequency division multiple access (OFDMA) in the uplink. We frame the self-coexistence problem as a non-cooperative game, and propose an uplink soft frequency reuse (USFR) technique to enable globally power-efficient and locally fair sharing of white-space spectrum. In each network, uplink resource allocation is decoupled into two subproblems: subchannel allocation (SCA) and transmit power control (TPC). We provide a unique optimal solution to the TPC subproblem, and present a low-complexity heuristic for the SCA subproblem. Furthermore, we frame the TPC and SCA games, and integrate them as a heuristic algorithm that achieves the Nash equilibrium in a fully distributed manner. Our simulation results show that the proposed USFR technique significantly improves self-coexistence in several aspects, including spectrum utilization, power consumption, and intra-cell fairness.**

I. Introduction

The opportunistic access to licensed spectrum by unlicensed secondary users equipped with cognitive radios (CRs) [1][2] and the resulting coexistence between secondary users and licensed primary users have given rise to a number of challenging technical problems. Thus far, the widely studied problem of primary-secondary network coexistence has been addressed by the use of incumbent geolocation databases [3][4] augmented with spectrum sensing techniques. In contrast, however, another important problem — the self-coexistence of secondary CR networks — has not yet been well understood.

A. Motivation

Recently, self-coexistence mechanisms have been proposed as a part of wireless standards for CR networks — e.g., IEEE 802.16h [5] and IEEE 802.22 [6]. For ease of implementation, however, most of these standardized self-coexistence mechanisms are conservative and can be inefficient. For instance, the self-coexistence protocols in IEEE 802.22 based on time division multiple access (TDMA) require that coexisting network cells do not occupy the same channel at the same time. In other words, *co-channel spectrum sharing* is not allowed. But when an insufficient number of channels are used to accommodate co-located CR networks operating in white-space spectrum, as expected in a dense urban environment, such co-channel sharing is unavoidable. In fact, it is possible that some of the user terminals in different cells share the same channel simultaneously with acceptable mutual interference. In many wireless standards including IEEE 802.16h and IEEE 802.22, orthogonal frequency division multiple access (OFDMA) has been widely used. In the OFDMA downlink, a soft frequency reuse (SFR) [7]-[11] is used to enable co-channel self-coexistence. However, the problem of achieving efficient co-channel sharing in the *uplink* has yet to be addressed adequately.

To fill the technical void, in this paper, we propose an uplink SFR (USFR) technique to enhance the self-coexistence of CR networks in the OFDMA uplink. However, designing such a technique is not trivial due to the following challenges:

First, USFR has to be spectrum-efficient in a dynamic environment. Unlike the downlink interference caused by relatively static broadcast signals from base stations (BSs), the uplink inter-network interference caused by user-generated signals is much more dynamic and unpredictable. The interference environment that USFR has to handle becomes even more complex when mobile/portable devices are considered in CR networks, e.g. IEEE 802.16 amended by 802.16h and IEEE 802.22a [12].

Second, USFR has to be carried out in a distributed manner. Unlike most licensed networks, e.g. cellular networks, that are deployed with careful central frequency planning, there is typically no central entity that can address the issues of spectrum sharing and inter-network interference for unlicensed CR networks. It is very probable that co-located coexisting networks operating in the same swath of spectrum are managed by different network operators. In most cases, reliable and real-time inter-network coordination cannot be available.

Third, USFR has to be globally power-efficient and locally fair. In view of potentially mobile/portable devices in CR networks, power saving is necessary for battery-powered uplink transmitters. In addition, fairness guarantee is also important, because user terminals, either close to or far away from their home BS, consume largely different amounts of power for the same level of signal to interference and noise ratio (SINR).

B. Contributions

The contributions of this paper are summarized as follows:

First, uplink resource allocation (URA) in each network cell is formulated as an optimization problem, and is further decoupled into two subproblems: subchannel allocation (SCA) and transmit power control (TPC). Solving the former one requires global knowledge, whereas solving the latter one does not.

Second, for the TPC subproblem, we provide a unique optimal solution. We frame multi-cell TPC as a non-cooperative

This work was supported in part by the National Science Foundation under grants CNS-0746925, CNS-0831865, and CNS-0910531, and the Institute for Critical Technology and Applied Science at Virginia Tech.

game, and prove that the Nash equilibrium can be established in the TPC game without inter-cell coordination.

Third, for the SCA subproblem requiring global knowledge, we present a low-complexity heuristic. We continue by framing multi-cell SCA as a non-cooperative game. After that, we integrate the TPC and SCA games, and formulate a two-level game-theoretic approach that is heuristic yet distributed.

Fourth, our simulation results show that the proposed USFR technique effectively enhances the self-coexistence of CR networks jointly in several aspects, including spectrum utilization, power consumption, and intra-cell fairness.

The remainder of this paper is organized as follows. Related work is discussed in section II. System framework and basic problem formulation are introduced in section III. Decoupled TPC and SCA subproblems are studied in sections IV and V, respectively. A two-level game-theoretic approach is proposed in section VI. Simulation results are presented in section VII, and conclusion is given in section VIII.

II. Related Work

There is already considerable existing work addressing the self-coexistence of OFDMA systems. In the downlink, several autonomous SFR-based mechanisms are proposed in [9]-[11]. However, there is very limited work applying SFR in the uplink. In [13], a semi-autonomous SFR-based resource allocation algorithm, which is similar to that in [9][10], is proposed to maximize uplink cell throughput. However, neither power consumption nor intra-cell fairness is properly studied, which should be more important in the uplink case. In [14], several heuristic SFR-based user scheduling mechanisms are roughly compared in terms of uplink outage probability. However, such mechanisms do not guarantee to create spectrum-efficient and power-efficient resource allocation patterns.

As for developing distributed mechanisms, non-cooperative game theory can be utilized to analyze the multi-cell resource allocation in both the downlink [15][16] and uplink [17][18], and the self-coexistence of CR networks [19][20]. However, there is no existing work targeting to jointly achieve spectrum utilization, power consumption, and intra-cell fairness.

III. System Model

In this section, we discuss the system model that underpins the proposed USFR technique as well as related issues.

A. Uplink Soft Frequency Reuse

In this paper, we assume a spectrum environment in a densely populated urban area, where co-channel spectrum sharing is needed to accommodate the demands of coexisting networks. In particular, we focus on the problem of sharing one single white-space channel among uncoordinated CR network cells, which are placed in adjacent geographic locations but managed by different network operators. The commonly shared channel further consists of a number of orthogonal subchannels. Each BS can allocate certain subcarriers on each uplink subchannel through OFDMA. User terminals associated with each BS, as their home BS, can be either fixed or mobile/portable. We do not address the issue of incumbent protection, as it is beyond the scope of this paper. The co-channel self-coexistence of CR networks can be enabled over one incumbent-free channel.

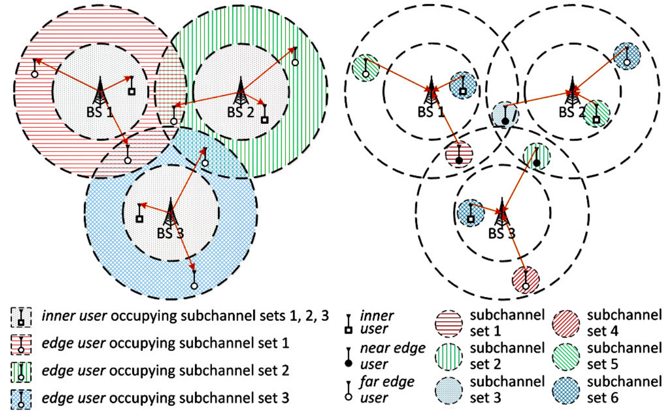

Fig. 1. Basic ideas: (a) SFR (left); (b) USFR (right).

The idea of SFR is illustrated in Fig. 1a. In each of the co-channel cells, the user terminals next to their home BS, called inner users, can fully occupy the entire common channel. But the user terminals far away from their home BS, called edge users, have to take an exclusive set of subchannels that should not be used in other cells. Clearly, the inner users in different cells can always access the same channel at the same time. Similarly, in Fig. 1b, the idea of USFR is to allow some users in a cell to share certain subchannels with some users in other cells. However, things are more complex in the uplink case. We still follow the same definitions of inner users and edge users. Furthermore, uplink edge users can be either *near* or *far* ones based on whether or not they are located close to any inter-cell overlapping areas. Hence, there can be a greater number of possible coexistence patterns, i.e. resource allocation patterns, created in the uplink by the combinations of inner users, near edge users, and far edge users operating in different cells.

B. Uplink Resource Allocation Problem

In each network cell, the BS is responsible for conducting uplink resource allocation (URA) for the users under control. Particularly, local URA includes subchannel allocation (SCA) and transmit power control (TPC) for the users' active uplink sessions. Whenever a new session becomes active in any cell, its home BS needs to redo URA in that cell to spare enough resource for the accommodation of this new session.

Suppose that there are total of N cells coexisting on a common channel, which consists of K subchannels. In each cell n for $n \in \mathcal{N} \triangleq \{1, \cdots, N\}$, there are $M^{(n)}$ active sessions. Each user in cell n can maintain multiple sessions, and each session m_n for $m_n \in \mathcal{M}^{(n)} \triangleq \{1, \cdots, M^{(n)}\}$ can operate on multiple subchannels. To avoid unnecessary intra-cell interference, each subchannel k for $k \in \mathcal{K} \triangleq \{1, \cdots, K\}$ cannot be assigned to more than one session in the same cell.

In the cell $n = a$, local URA strategy is characterized by SCA and TPC strategy matrices, which are denoted by $\mathbf{U}^{(a)} \triangleq \{U^{(a)}_{m_a,k}\}_{M^{(a)} \times K}$ and $\mathbf{P}^{(a)} \triangleq \{P^{(a)}_{m_a,k}\}_{M^{(a)} \times K}$, respectively. Here, each $U^{(a)}_{m_a,k}$ in $\mathcal{U}^{(a)} \triangleq \{\mathbf{U}^{(a)}\}$ denotes a binary indicator that is equal to '1' ('0') when session m_a takes (does not take) subchannel k, and each $P^{(a)}_{m_a,k}$ in $\mathcal{P}^{(a)} \triangleq \{\mathbf{P}^{(a)}\}$ denotes the corresponding allocated power level. The set of URA strategies in the other $N-1$ cells is characterized by strategy matrix sets $\mathcal{U}^{-a} \triangleq \dot{\times}_{n \in \mathcal{N}, n \neq a} \mathcal{U}^{(n)}$ and $\mathcal{P}^{-a} \triangleq \dot{\times}_{n \in \mathcal{N}, n \neq a} \mathcal{P}^{(n)}$, where the operator $\dot{\times}$ represents the Cartesian product [21].

Ideally, the local URA in the cell a can be formulated as a global optimization problem. Note that the assumption of perfect global knowledge will be removed later. The cell a's local objective in terms of power consumption, $L^{(a)}$, is defined as

$$L^{(a)} \triangleq \sum_{m_a=1}^{M^{(a)}} w_{m_a}^{(a)} \sum_{k=1}^{K} P_{m_a,k}^{(a)}, \tag{1}$$

where each $w_{m_a}^{(a)}$ denotes session m_a's weight or priority. In consideration of the intra-cell fairness among inner users, near edge users, and far edge users, we define $w_{m_a}^{(a)}$ as

$$w_{m_a}^{(a)} \triangleq \frac{\sum_{n=1,n\neq a}^{N} H_{m_a}^{(a,n)}}{H_{m_a}^{(a,a)}}, \tag{2}$$

where each $H_{m_a}^{(a,n)}$ denotes the propagation gain from session m_a to BS n. Under such definition of user weights, near edge users are mostly assigned with the highest priorities for power saving; inner users are assigned with the lowest priorities for interference-free subchannels; and far edge users' priorities are somewhere in between. This setting is based on the facts that near edge users always have to waste significant power for fading and shadowing, and they are likely to cause the greatest harmful interference to other cells in the uplink.

There are several constraints for the minimization of $L^{(a)}$:

First, if $U_{m_a,k}^{(a)} = 1$, then $P_{m_a,k}^{(a)}$ should be lower bounded by $\hat{Q}_{m_a,k}^{(a)}$, the minimum power for meeting session m_a's SINR requirement, denoted by $\gamma_{m_a}^{(a)}$. Moreover, $P_{m_a,k}^{(a)}$ should also be upper bounded by $\bar{Q}_{m_a,k}^{(a)}$, the maximum power of session m_a on each subchannel k. But if $U_{m_a,k}^{(a)} = 0$, then $P_{m_a,k}^{(a)} = 0$. The BS a cannot make decisions for the other cells to change $\mathcal{U}^{-a}$, so we say $\mathcal{U}^{-a} \equiv \tilde{\mathcal{U}}^{-a}$, where $\tilde{\mathcal{U}}^{-a}$ is a fixed strategy matrix set. But $\mathcal{P}^{(a)}$ and $\mathcal{P}^{-a}$ may interact with each other due to the change of inter-cell interference. Hence, these bounds should be satisfied in each cell n, and we have

$$\begin{gathered} U_{m_n,k}^{(n)} \hat{Q}_{m_n,k}^{(n)} \leq P_{m_n,k}^{(n)} \leq U_{m_n,k}^{(n)} \bar{Q}_{m_n,k}^{(n)} \\ \text{for } n \in \mathcal{N}; m_n \in \mathcal{M}^{(n)}; k \in \mathcal{K}, \end{gathered} \tag{3}$$

where each $\hat{Q}_{m_n,k}^{(n)}$ is written as

$$\hat{Q}_{m_n,k}^{(n)} \triangleq \frac{\gamma_{m_n}^{(n)} (\sum_{n'=1,n'\neq n}^{N} \sum_{m_{n'}=1}^{M^{(n')}} P_{m_{n'},k}^{(n')} H_{m_{n'}}^{(n',n)} + N_0)}{H_{m_n}^{(n,n)}},$$

in which N_0 denotes average noise power. At each BS n, the interference measurement on subchannel k is written as

$$I_k^{(n)} \triangleq \sum_{n'=1,n'\neq n}^{N} \sum_{m_{n'}=1}^{M^{(n')}} P_{m_{n'},k}^{(n')} H_{m_{n'}}^{(n',n)}.$$

Second, each session m_n's aggregated uplink capacity (per unit bandwidth) in cell n should meet its corresponding QoS requirement, denoted by $\theta_{m_n}^{(n)}$, and we have

$$\begin{gathered} \sum_{k=1}^{K} \log(1 + \frac{P_{m_n,k}^{(n)} H_{m_n}^{(n,n)}}{I_k^{(n)} + N_0}) \geq \theta_{m_n}^{(n)} \\ \text{for } n \in \mathcal{N}; m_n \in \mathcal{M}^{(n)}. \end{gathered} \tag{4}$$

Third, as above, the BS a should not assign more than one session in the same cell to any subchannel k, and we have

$$\sum_{m_a=1}^{M^{(a)}} U_{m_a,k}^{(a)} \leq 1 \text{ for } k \in \mathcal{K}. \tag{5}$$

The URA problem in the cell a towards our goal of globally power-efficient and locally fair USFR is defined as follows.

Problem 1 (URA) Find: $\mathcal{U}^{(a)}, \mathcal{P}^{(a)}, \mathcal{P}^{-a}$;
Minimize: $L^{(a)}$;
Subject to: (3), (4), (5).

The Problem 1 is formulated as a mixed-integer non-linear program (MINLP), which is NP-hard in general. Therefore, solving it directly can be costly. As in [15], we can decouple the complex URA problem in the cell a into two subproblems:

- SCA by adapting $\mathcal{U}^{(a)}$ given fixed $\mathcal{P}^{(a)}$ and $\mathcal{P}^{-a}$;
- TPC by adapting $\mathcal{P}^{(a)}$ and $\mathcal{P}^{-a}$ given fixed $\mathcal{U}^{(a)}$.

In sections IV and V, these two subproblems will be studied in detail in each cell and in the entire N-cell system.

C. A Game-Theoretic Framework

Due to the distributed nature of CR networks, each cell in the multi-cell system has to conduct local URA individually. In view of possible conflicts in coexisting cells' local optimal strategies, we choose to make use of game theory to study the global performance of multi-cell URA problem.

The self-coexistence of uncoordinated CR networks can be modeled as a non-cooperative game, in which each network cell acts as a player. In the URA game, each cell solves Problem 1 independently. Then, minimizing $L^{(n)}$ is equivalent to optimizing cell n's utility. According to the decoupled SCA and TPC subproblems, we adopt a two-level game-theoretic approach to generate globally power-efficient and locally fair coexistence patterns in a distributed manner. Specifically, the URA game can be regarded as two levels of non-cooperative games for SCA and TPC, respectively. In the two-level framework, each acting cell plays the SCA game on the first level. Given a strategy taken by any cell in the SCA game, the Nash equilibrium is achieved in the TPC game on the second level, which will be proved in the next section. The optimal utility gain by taking this SCA strategy is shown accordingly. Based on the utility gain, the acting cell is able to know whether this two-level URA strategy is beneficial. As soon as nobody can find an improving strategy, a stabilized coexistence pattern is commonly agreed by all the cells in the URA game.

IV. Game for Transmit Power Control

In this section, we focus on the TPC subproblem, and study multi-cell TPC as a non-cooperative game. The optimal TPC strategy in each cell is proved to optimize the sessions' power consumption as well as power efficiency. Moreover, the Nash equilibrium is proved to be established in the TPC game.

A. Transmit Power Control Subproblem

The existence of binary variables in $\mathcal{U}^{(a)}$ makes Problem 1 costly to solve. If $\mathcal{U}^{(a)}$ is fixed in advance, then the URA problem can be reduced to the TPC subproblem. The optimal TPC strategy set, say $\tilde{\mathcal{P}}^{N}$, to it depends on the setting of $\mathcal{U}^{(a)}$, say

$\mathcal{U}^{(a)} \equiv \tilde{\mathcal{U}}^{(a)}$. In our two-level framework, $\tilde{\mathcal{U}}^{(a)}$ comes from the previously solved SCA subproblem. Thus, $\tilde{\mathcal{U}}^{(a)}$ and $\tilde{\mathcal{P}}^{N}$ together should satisfy the constraints of Problem 1. In addition to (3), (4), and (5) for $a \in \mathcal{N}$, the setting of $\mathcal{U}^{(a)} \equiv \tilde{\mathcal{U}}^{(a)}$ also needs to satisfy the conditions (6) and (8) below to make $\tilde{\mathcal{P}}^{N}$ feasible to Problem 1. We have

$$U^{(n)}_{m_n,k}\tilde{Q}^{(n)}_{m_n,k} \leq U^{(n)}_{m_n,k}\bar{Q}^{(n)}_{m_n,k} \quad \text{for } n \in \mathcal{N}; m_n \in \mathcal{M}^{(n)}; k \in \mathcal{K}, \tag{6}$$

where each $\tilde{Q}^{(n)}_{m_n,k} = \hat{Q}^{(n)}_{m_n,k}|_{P^{(n')}_{m_{n'},k}=\tilde{P}^{(n')}_{m_{n'},k}}$ and each $\tilde{P}^{(n)}_{m_n,k}$ is from the solution to the following system of equations

$$P^{(n)}_{m_n,k} \equiv U^{(n)}_{m_n,k}\hat{Q}^{(n)}_{m_n,k} \text{ for } n \in \mathcal{N}; m_n \in \mathcal{M}^{(n)}; k \in \mathcal{K}. \tag{7}$$

Note that the values of $\tilde{P}^{(n)}_{m_n,k}$ are determined as long as $\tilde{\mathcal{U}}^{(a)}$ is fixed, and so are that of $\tilde{Q}^{(n)}_{m_n,k}$. Each $\tilde{Q}^{(n)}_{m_n,k}$ is the minimum possible value of $\hat{Q}^{(n)}_{m_n,k}$ under $\mathcal{U}^{(a)} \equiv \tilde{\mathcal{U}}^{(a)}$. We also have

$$\sum_{k=1}^{K} \log(1 + U^{(n)}_{m_n,k}\gamma^{(n)}_{m_n}) \geq \theta^{(n)}_{m_n} \quad \text{for } n \in \mathcal{N}; m_n \in \mathcal{M}^{(n)}. \tag{8}$$

Hence, the TPC subproblem in the cell a is defined as follows.

Problem 2 (TPC) Find: $\mathcal{P}^{(a)}, \mathcal{P}^{-a}$;
Minimize: $L^{(a)}$;
Subject to: (3), (4), (5), (6), (8).

Lemma 1 Given that the previously fixed SCA strategy set $\tilde{\mathcal{U}}^{N} = \tilde{\mathcal{U}}^{(a)}\dot{\times}\tilde{\mathcal{U}}^{-a}$ satisfies (5) for $a \in \mathcal{N}$, (6), and (8), the unique optimal solution of TPC strategy set, say $\tilde{\mathcal{P}}^{N} = \tilde{\mathcal{P}}^{(a)}\dot{\times}\tilde{\mathcal{P}}^{-a}$, to Problem 2 satisfies (7).

Proof According to (1) and (2), we know that

$$\frac{\partial L^{(a)}}{\partial P^{(a)}_{m_a,k}} = w^{(a)}_{m_a} > 0 \text{ for } m_a \in \mathcal{M}^{(a)}; k \in \mathcal{K}. \tag{9}$$

Obviously, in order to minimize $L^{(a)}$, each $P^{(a)}_{m_a,k}$ in $\mathcal{P}^{(a)}$ has to be as small as possible. Then, we can see that any inequality relationship $P^{(a)}_{m_a,k} \geq U^{(a)}_{m_a,k}\hat{Q}^{(a)}_{m_a,k}$ in (3) can be rewritten as

$$P^{(a)}_{m_a,k} \geq \sum_{n=1,n\neq a}^{N} \sum_{m_n=1}^{M^{(n)}} c^{(n,a)}_{m_n,m_a,k}P^{(n)}_{m_n,k} + c^{(a)}_{m_a,k} \quad \text{for } m_a \in \mathcal{M}^{(a)}; k \in \mathcal{K}, \tag{10}$$

in which all the coefficients c's are non-negative. Furthermore, for each $P^{(n)}_{m_n,k}$ on the right-hand side (RHS) of (10), we have

$$P^{(n)}_{m_n,k} \geq \sum_{n'=1,n'\neq n}^{N} \sum_{m_{n'}=1}^{M^{(n')}} c^{(n',n)}_{m_{n'},m_n,k}P^{(n')}_{m_{n'},k} + c^{(n)}_{m_n,k} \quad \text{for } n \in \mathcal{N}, n \neq a; m_n \in \mathcal{M}^{(n)}; k \in \mathcal{K}, \tag{11}$$

in which all the coefficients c's are non-negative. We can see that every $P^{(n)}_{m_n,k}$ in $\mathcal{P}^{(n)}$ is lower bounded by a linear combination of $P^{(n')}_{m_{n'},k}$ in $\mathcal{P}^{-n}$ with all non-negative coefficients. All the inequality relationships in (10) and (11) are in a cycle, since each $P^{(a)}_{m_a,k}$ on the left-hand side (LHS) of (10) appears on the RHS of (11) as well. Clearly, when the equalities hold for all in (10) and (11), each $P^{(n)}_{m_n,k}$ in $\mathcal{P}^{N} = \mathcal{P}^{(a)}\dot{\times}\mathcal{P}^{-a}$ reaches its lower bound $U^{(n)}_{m_n,k}\hat{Q}^{(n)}_{m_n,k} = U^{(n)}_{m_n,k}\tilde{Q}^{(n)}_{m_n,k}$. At the same time, the weighted sum $L^{(a)}$ is minimized by $\tilde{\mathcal{P}}^{N}$ that solves (7) without considering any other constraints. Note that (7) is a system of linear equations, thus $\tilde{\mathcal{P}}^{N}$ is unique.

We next verify the feasibility of $\tilde{\mathcal{P}}^{N}$, which is the unique solution that minimizes $L^{(a)}$. As long as $\tilde{\mathcal{U}}^{N}$ and $\tilde{\mathcal{P}}^{N}$ together satisfy the constraints of Problem 1, we get the desired optimal TPC strategy set. It is easy to see that (3) holds given (6) and (7). If (7) holds, the LHSs of (4) and (8) are same, so (4) holds given (7) and (8). And (5) for $a \in \mathcal{N}$ is known to hold. Hence, $\tilde{\mathcal{P}}^{N}$ satisfies the constraints of Problem 1, and is thus the unique optimal solution to Problem 2. □

In addition to minimizing the sessions' power consumption as defined by $L^{(a)}$, we are also interested in maximizing the sessions' power efficiency, i.e. uplink capacity per unit power. Again in the form of weighted sum, the cell a's local objective in terms of power efficiency, $E^{(a)}$, is defined as

$$E^{(a)} \triangleq \sum_{m_a=1}^{M^{(a)}} v^{(a)}_{m_a} \sum_{\substack{k=1 \\ U^{(a)}_{m_a,k}\neq 0}}^{K} \frac{\log(1 + \frac{P^{(a)}_{m_a,k}H^{(a,a)}_{m_a}}{I^{(a)}_k+N_0})}{P^{(a)}_{m_a,k}}, \tag{12}$$

where each $v^{(a)}_{m_a}$ can be any positive weight. At the BS a, the SINR measurement on subchannel k is written as

$$S^{(a)}_{m_a,k} \triangleq \frac{P^{(a)}_{m_a,k}H^{(a,a)}_{m_a}}{I^{(a)}_k + N_0}.$$

Lemma 2 The unique solution of TPC strategy set that satisfies (7), i.e. $\tilde{\mathcal{P}}^{N} = \tilde{\mathcal{P}}^{(a)}\dot{\times}\tilde{\mathcal{P}}^{-a}$, maximizes $E^{(a)}$, subject to the constraints of Problem 2.

Proof We start from proving that $\tilde{\mathcal{P}}^{N}$ maximizes

$$\hat{E}^{(a)} \triangleq \sum_{m_a=1}^{M^{(a)}} v^{(a)}_{m_a} \sum_{\substack{k=1 \\ U^{(a)}_{m_a,k}\neq 0}}^{K} \frac{\log(1 + S^{(a)}_{m_a,k})}{S^{(a)}_{m_a,k}}. \tag{13}$$

Define $f(s) = \frac{s}{1+s} - \log(1+s)$. It is trivial to prove that $f(s)$ is a non-increasing function of s. Hence, $f(S^{(a)}_{m_a,k}) \leq f(0) \equiv 0$, since each $S^{(a)}_{m_a,k} \geq 0$ always holds. Then, we have

$$\frac{\partial \hat{E}^{(a)}}{\partial S^{(a)}_{m_a,k}} = v^{(a)}_{m_a} \frac{\frac{S^{(a)}_{m_a,k}}{1+S^{(a)}_{m_a,k}} - \log(1 + S^{(a)}_{m_a,k})}{(S^{(a)}_{m_a,k})^2} \leq 0 \quad \text{for } m_a \in \mathcal{M}^{(a)}; k \in \mathcal{K}. \tag{14}$$

As in the proof for Lemma 1, each $S^{(a)}_{m_a,k}$ is minimized to $U^{(a)}_{m_a,k}\gamma^{(a)}_{m_a}$ by $\tilde{\mathcal{P}}^{N}$. And $\hat{E}^{(a)}$ is maximized accordingly. To find the relationship between $E^{(a)}$ and $\hat{E}^{(a)}$, we can see that

$$\frac{\log(1 + S^{(a)}_{m_a,k})}{P^{(a)}_{m_a,k}} = \frac{\log(1 + S^{(a)}_{m_a,k})}{S^{(a)}_{m_a,k}} \frac{H^{(a,a)}_{m_a}}{I^{(a)}_k + N_0} \quad \text{for } m_a \in \mathcal{M}^{(a)}; k \in \mathcal{K}. \tag{15}$$

On the RHS of (15), $\tilde{\mathcal{P}}^N$ maximizes each $\frac{\log(1+S^{(a)}_{m_a,k})}{S^{(a)}_{m_a,k}}$, and the corresponding $I^{(a)}_k$ is minimized simultaneously. Hence, each $\frac{\log(1+S^{(a)}_{m_a,k})}{P^{(a)}_{m_a,k}}$ is maximized by $\tilde{\mathcal{P}}^N$ that solves (7), and so is the weighted sum $E^{(a)}$ with all positive weights. □

B. Multi-Cell TPC as Non-Cooperative Game

Although the optimality of the TPC subproblem is proved to be attainable, the behavior of each cell in presence of other coexisting cells is still unclear. We frame multi-cell TPC problem as a non-cooperative game, which serves as the second level of the URA game as in our two-level framework. In the TPC game, each cell solves Problem 2 independently.

Theorem 1 The unique solution of TPC strategy set that satisfies (7), i.e. $\tilde{\mathcal{P}}^N$, establishes the Nash equilibrium in the non-cooperative TPC game defined by Problem 2.

Proof According to the fixed point theorem [21] in game theory, two conditions must be satisfied for the existence of Nash equilibrium in the non-cooperative game:

- The strategy space, $\mathbb{P}^N = \dot{\times}_{n\in\mathcal{N}}\mathbb{P}^{(n)}$, for searching $\mathcal{P}^N$ should be a non-empty, compact, and convex subset of certain Euclidean space;
- The utility functions, $L^{(n)}$ for $n \in \mathcal{N}$, should be continuous in $\mathcal{P}^N$ and quasi-convex in $\mathcal{P}^{(n)}$.

Due to the bounds defined in (3) for each $P^{(n)}_{m_n,k}$ in $\mathcal{P}^{(n)}$, $\mathbb{P}^{(n)}$ in each cell n is closed and bounded, and thus compact. It is also trivial to show that $\mathbb{P}^{(n)}$ is convex. We know that the Cartesian product of compact and convex sets is still compact and convex. Hence, the first condition is satisfied. As defined in (1), each $L^{(n)}$ is a linear combination of $P^{(n)}_{m_n,k}$ in $\mathcal{P}^{(n)}$ with all positive coefficients. Clearly, $L^{(n)}$ is continuous in $\mathcal{P}^N$. We know that a linear combination of convex functions with positive coefficients is again convex, and every convex function is also quasi-convex. Then, $L^{(n)}$ is quasi-convex in $\mathcal{P}^{(n)}$. Hence, the second condition is satisfied as well.

As in the proof for Lemma 1, not only $P^{(a)}_{m_a,k}$ in $\mathcal{P}^{(a)}$ but also $P^{(n)}_{m_n,k}$ in $\mathcal{P}^{-a}$ are minimized to $U^{(n)}_{m_n,k}\tilde{Q}^{(n)}_{m_n,k}$ by $\tilde{\mathcal{P}}^N$. These lower bounds are determined as soon as $\mathcal{U}^N$ is fixed. Regardless the different positive weights in coexisting cells' local objectives, the optimal TPC strategies applied in different cells are always the same unique one, i.e. $\tilde{\mathcal{P}}^N = \tilde{\mathcal{P}}^{(1)}\dot{\times}\tilde{\mathcal{P}}^{-1} = \cdots = \tilde{\mathcal{P}}^{(N)}\dot{\times}\tilde{\mathcal{P}}^{-N}$, that solves (7). According to the fixed point theorem, we know that any fixed point of the best strategy space, $\mathbb{R}^N = \dot{\times}_{n\in\mathcal{N}}\mathbb{R}^{(n)}$, is the Nash equilibrium. Surely, the unique common best strategy set $\tilde{\mathcal{P}}^N$ is a fixed point of $\mathbb{R}^N$ in the TPC game, since any cell's best strategy is always to stay once $\tilde{\mathcal{P}}^N$ has been reached. As a result, the Nash equilibrium is established by $\tilde{\mathcal{P}}^N$ in the TPC game. □

The Theorem 1 offers a guideline of designing TPC algorithms in a multi-cell system. In fact, there are already various existing TPC algorithms that do not rely on perfect inter-cell coordination, e.g. iterative water-filling algorithm [15]. Given each fixed SCA strategy set, multi-cell TPC can be performed iteratively in a distributed manner. Specifically, in each cell n, any $P^{(n)}_{m_n,k}$ with $U^{(n)}_{m_n,k} = 1$ is gradually increased from zero following the rise of $I^{(n)}_k$ to always just keep $S^{(n)}_{m_n,k} \equiv \gamma^{(n)}_{m_n}$. If there exists a solution to (7), TPC is done when the SINR requirements of all the sessions have been met.

V. Game for Subchannel Allocation

In this section, we focus on the SCA subproblem, and study multi-cell SCA as a non-cooperative game as well. However, unlike the case of TPC that can be addressed optimally, several undesired facts for the SCA subproblem and SCA game are briefly discussed before going any further.

A. Subchannel Allocation Subproblem

If $\mathcal{P}^{(a)}$ and $\mathcal{P}^{-a}$ in Problem 1 are fixed in advance, then the URA problem can be reduced to the SCA subproblem. Taking advantage of the unique optimal solution to Problem 2 as in Lemmas 1 and 2, the setting of $\mathcal{P}^{(a)}$ and $\mathcal{P}^{-a}$ can be $\mathcal{P}^N = \mathcal{P}^{(a)}\dot{\times}\mathcal{P}^{-a} \equiv \tilde{\mathcal{P}}^N$. Besides (7) for such setting, the conditions (5) for $a \in \mathcal{N}$, (6), and (8) for the setting of $\mathcal{U}^{(a)} \equiv \tilde{\mathcal{U}}^{(a)}$ in Problem 2 also become the constraints of the SCA subproblem in the cell a, which is defined as follows.

Problem 3 (SCA)	Find:	$\mathcal{U}^{(a)}$;
	Minimize:	$L^{(a)}$;
	Subject to:	(5), (6), (7), (8).

In fact, solving Problem 3 is equivalent to performing both the two levels of URA in the cell a, i.e. SCA and TPC. As in Theorem 1, the setting of $\mathcal{P}^N \equiv \tilde{\mathcal{P}}^N$ in Problem 3 is based on the Nash equilibrium established in the TPC game defined by Problem 2. In our two-level framework, given each SCA strategy, say $\tilde{\mathcal{U}}^{(a)}$, taken by the cell a on the first level, the TPC game is played on the second level and gives $\tilde{\mathcal{P}}^N$ under $\mathcal{U}^{(a)} \equiv \tilde{\mathcal{U}}^{(a)}$. Thus, Problem 2 always follows Problem 3.

B. Multi-Cell SCA as Non-Cooperative Game

Like the TPC game, we try to frame multi-cell SCA problem as a non-cooperative game, where each cell solves Problem 3 independently. However, the SCA game that serves as the first level of the URA game does not have the desirable properties as the TPC game does due to the following facts.

Corollary 1 Solving Problem 3 for the optimal SCA strategy necessarily requires the perfect global knowledge of $\tilde{\mathcal{U}}^{-a}$, $\tilde{\mathcal{P}}^{-a}$, and $H^{(n,n')}_{m_n}$ for $n \in \mathcal{N}, n \neq a$ in the cell a.

Proof Obviously, real-time inter-cell coordination is necessary to keep track of $\tilde{\mathcal{U}}^{-a}$ and $\tilde{\mathcal{P}}^{-a}$. Although it is possible to estimate the values of $H^{(a,n')}_{m_a}$ in the cell a by decoding downlink pilot signals from BSs n' [13][14], that of $H^{(n,n')}_{m_n}$ for $n \neq a$ are hard to get without inter-cell coordination. □

Corollary 2 The worst-case complexity of searching for the optimal solution to Problem 3 is $O(K!)$.

Proof Due to the setting of $\mathcal{P}^N \equiv \tilde{\mathcal{P}}^N$, any session m_a's uplink capacity on each taken subchannel is always $\log(1 + \gamma^{(a)}_{m_a})$ given $\gamma^{(a)}_{m_a}$. To minimize $L^{(a)}$ and make (8) hold, the number of subchannels taken by each session m_a is given by

$$T^{(a)}_{m_a} \triangleq \lceil \frac{\theta^{(a)}_{m_a}}{\log(1 + \gamma^{(a)}_{m_a})} \rceil.$$

Then, the number of subchannels taken by the cell a is $T^{(a)} = \sum_{m_a=1}^{M^{(a)}} T^{(a)}_{m_a}$. In view of (5), the number of possible solutions for $\mathcal{U}^{(a)}$ is $\frac{K!}{(K-T^{(a)})!}$. Hence, the complexity of solving Problem 3 via brute force is on the order of $O(K!)$. ☐

Corollary 3 The Nash equilibrium does not always exist in the non-cooperative SCA game defined by Problem 3.

Proof The discrete strategy space for searching $\mathcal{U}^{(a)}$ cannot satisfy the conditions for the existence of Nash equilibrium as in the proof for Theorem 1. The endless cycles representing interest conflicts among coexisting cells may occur. ☐

VI. A Two-Level Game-Theoretic Approach

Towards the globally power-efficient and locally fair USFR, we propose a two-level game-theoretic approach. In general, the two-level approach is still based on Problems 2 and 3. But some extra effort is necessary due to the above facts. Instead of getting the optimal solution relying on perfect global knowledge and high computational capability, we resort to a heuristic yet practical approach. In consideration of uncoordinated CR networks coexisting in dynamic white-space spectrum, the proposed low-complexity algorithm can converge to near-optimal coexistence patterns in a fully distributed manner.

A. Local Uplink Resource Allocation Algorithm

Without assuming perfect global knowledge and high computational capability, local SCA in each cell can be conducted heuristically. In the cell a, solving Problem 3 is to find $\tilde{\mathcal{U}}^{(a)}$ that minimizes the weighted sum of sessions' power consumption. The original purpose of defining user weights is to achieve intra-cell fairness. In (2), the numerator $\sum_{n=1,n\neq a}^{N} H_{m_a}^{(a,n)}$ characterizes session m_a's potential of interference *to* other cells, while the denominator $H_{m_a}^{(a,a)}$ characterizes session m_a's tolerance for interference *from* other cells. The sessions with larger weights are either more likely to cause interference or more vulnerable to interference. Thus, such weights representing the sessions' priorities can be used to heuristically perform local SCA. Intuitively, near edge users tend to claim dedicated subchannels for complete interference avoidance and need to be scheduled at first, while in contrast, inner users are most coexistence-friendly and can be scheduled at last.

Now, we focus on a certain session, say m_a, that has been scheduled to take subchannels in the cell a. Suppose that the perfect knowledge of $\tilde{\mathcal{U}}^{-a}$, $\tilde{\mathcal{P}}^{-a}$, and $H_{m_n}^{(n,a)}$ for $n \neq a$ is not available. The only information that can be locally obtained by the BS a is $I_k^{(a)}$, i.e. the aggregated interference measurement on each subchannel k. Hence, the local SCA decisions for the session m_a is merely based on the interference measurements $\hat{I}_k^{(a)} \triangleq I_k^{(a)}(\tilde{\mathcal{P}}^N|_{U_{m_a,k}^{(a)}=0})$ or $\bar{I}_k^{(a)} \triangleq I_k^{(a)}(\tilde{\mathcal{P}}^N|_{U_{m_a,k}^{(a)}=1})$, where $\tilde{\mathcal{P}}^N$ is generated in the TPC game under the fixed $\tilde{\mathcal{U}}^{-a}$ and certain $\mathcal{U}^{(a)}$. Intuitively, the session m_a is supposed to take $T_{m_a}^{(a)}$ subchannels with the lowest $\bar{I}_k^{(a)}$ to meet the required SINR and QoS. However, the values of $\bar{I}_k^{(a)}$ are costly to get if inter-cell coordination is not assumed, since the session m_a has to try each subchannel k by triggering the TPC game to create $\bar{I}_k^{(a)}$. Instead, the session m_a can take $T_{m_a}^{(a)}$ subchannels with the lowest $\hat{I}_k^{(a)}$ thanks to the connection between $\hat{I}_k^{(a)}$ and $\bar{I}_k^{(a)}$. In most cases, $\bar{I}_{k_1}^{(a)} < \bar{I}_{k_2}^{(a)}$ is true if $\hat{I}_{k_1}^{(a)} < \hat{I}_{k_2}^{(a)}$ for the same session m_a. Therefore, a heuristic solution of SCA strategy to Problem 3 can be given by Algorithm 1 as follows.

Algorithm 1 Local URA algorithm

1: **sort** the sequence of session m_a for $m_a \in \mathcal{M}^{(a)}$ by $w_{m_a}^{(a)}$ in descending order, and store the sorted sequence in array $\mathbf{M}(i)$ for $i = 1 \rightarrow M^{(a)}$
2: **set** $\mathcal{U}^{(a)} = \{\mathbf{0}\}$, and measure $\hat{I}_k^{(a)}$ for $k \in \mathcal{K}$ generated in the TPC game
3: **sort** the sequence of subchannel k for $k \in \mathcal{K}$ by $\hat{I}_k^{(a)}$ in ascending order, and store the sorted sequence in array $\mathbf{K}(j)$ for $j = 1 \rightarrow K$
4: **for** $i = 1 \rightarrow M^{(a)}$ **do**
5:   **set** $m_a = \mathbf{M}(i)$
6:   **for** $j = 1 \rightarrow K$ **do**
7:     **set** $k = \mathbf{K}(j)$
8:     **if** $\sum_{m=1}^{M^{(a)}} U_{m,k}^{(a)} == 0$ **then**
9:       **set** $U_{m_a,k}^{(a)} = 1, \cdots, U_{m_a,k+T_{m_a}^{(a)}-1}^{(a)} = 1$
10:       **break**
11:     **end if**
12:   **end for**
13: **end for**
14: **measure** $\bar{I}_k^{(a)}$ for k whose $\sum_{m=1}^{M^{(a)}} U_{m,k}^{(a)} == 1$ generated in the TPC game, and adjust $P_{m_a,k}^{(a)}$ if $U_{m_a,k}^{(a)} == 1$

Corollary 4 The worst-case complexity of running Algorithm 1 is $O(KM^{(a)})$.

Proof The major cost for running Algorithm 1 comes from the two sorting operations in lines 1 and 3 and the nested FOR loops between lines 4 and 13. Sorting sessions leads to the complexity of $O(M^{(a)} \log M^{(a)})$ via e.g. merge sort. Likewise, sorting subchannels incurs $O(K \log K)$. The nested FOR loops result in the complexity of $O(KM^{(a)})$. Because $K > M^{(a)}$ and possibly $M^{(a)} > \log K$, the complexity of running Algorithm 1 is on the order of $O(KM^{(a)})$. ☐

B. Two-Level Game-Theoretic Algorithm

Due to the fact that the SCA game as the first level of the non-cooperative URA game does not always converge to the Nash equilibrium, a cost or pricing function [21] should be considered in the definition of utility. Although our goal is to define a cost function that does not require global knowledge, we can still get some clues from a cooperative URA game where coexisting cells share the same global objective. Any cell's common global objective, G, can be defined as

$$G \triangleq \sum_{n=1}^{N} L^{(n)} = \sum_{n=1}^{N} \sum_{m_n=1}^{M^{(n)}} w_{m_n}^{(n)} \sum_{k=1}^{K} P_{m_n,k}^{(n)}. \tag{16}$$

It is trivial to show that the cooperative game in which each cell minimizes G can converge to the unique globally optimal coexistence pattern. In the end, nobody in the game can further decrease the value of G. We can rewrite (16) as $G = L^{(a)} + C_g^{(a)}$, where $C_g^{(a)} = \sum_{n=1,n\neq a}^{N} L^{(n)}$ is equivalent to the cell a's global cost function and can be rewritten as

$$C_g^{(a)} \triangleq \sum_{n=1,n\neq a}^{N} \sum_{m_n=1}^{M^{(n)}} \sum_{n'=1,n'\neq n}^{N} \sum_{k=1}^{K} \frac{P_{m_n,k}^{(n)} H_{m_n}^{(n,n')}}{H_{m_n}^{(n,n)}}. \tag{17}$$

On the RHS of (17), each $P^{(n)}_{m_n,k}H^{(n,n')}_{m_n}$ represents the interference from session m_n to BS n'. In the cell a, however, only the sum of the interference components $P^{(n)}_{m_n,k}H^{(n,a)}_{m_n}$ on each subchannel k, i.e. $I^{(a)}_k$, is known. Hence, the cell a's local cost function, denoted by $C^{(a)}_l$, can be defined as

$$C^{(a)}_l \triangleq \delta^{(a)} \sum_{k=1}^{K} I^{(a)}_k, \tag{18}$$

where $\delta^{(a)}$ denotes a positive price factor that is adaptable. Note that unlike the commonly defined cost functions, such as $\sum_{m_a=1}^{M^{(a)}} \sum_{k=1}^{K} \log(P^{(a)}_{m_a,k})$ [21], that change in the opposite direction of the change in $L^{(a)}$ as any $P^{(a)}_{m_a,k}$ is adapted, $C^{(a)}_l$ and $L^{(a)}$ can be reduced together most of the time. This is because the physical meaning of $L^{(a)}$ is the weighted sum of interference *to* other cells, and that of $C^{(a)}_l$ is the weighted sum of interference *from* other cells. When the cell a performs local URA to minimize $L^{(a)}$, most of the coexisting cells in the multi-cell system can reduce power simultaneously for the drop of interference from the cell a. Based on such heuristic, globally power-efficient self-coexistence can be achievable.

After replacing $L^{(a)}$ with $L^{(a)} + C^{(a)}_l$, the revised Problem 1 is again decoupled into two subproblems: SCA and TPC. The revised TPC subproblem in the cell a is stated as follows.

Problem 4 (TPC) Find: $\mathcal{P}^{(a)}, \mathcal{P}^{-a}$;
Minimize: $L^{(a)} + C^{(a)}_l$;
Subject to: (3), (4), (5), (6), (8).

Lemma 3 Given that the previously fixed SCA strategy set $\tilde{\mathcal{U}}^N = \tilde{\mathcal{U}}^{(a)} \dot{\times} \tilde{\mathcal{U}}^{-a}$ satisfies (5) for $a \in \mathcal{N}$, (6), and (8), the unique optimal solution of TPC strategy set, say $\tilde{\mathcal{P}}^N = \tilde{\mathcal{P}}^{(a)} \dot{\times} \tilde{\mathcal{P}}^{-a}$, to Problem 4 satisfies (7). Globally, $\tilde{\mathcal{P}}^N$ further establishes the Nash equilibrium in the non-cooperative TPC game defined by Problem 4.

Proof The proof follows the same logic as that for Lemma 1 and Theorem 1, and thus is omitted. □

As above, taking advantage of the unique optimal solution to Problem 4 as in Lemma 3, the setting of $\mathcal{P}^N \equiv \tilde{\mathcal{P}}^N$ gives the revised SCA subproblem in the cell a as follows.

Problem 5 (SCA) Find: $\mathcal{U}^{(a)}$;
Minimize: $L^{(a)} + C^{(a)}_l$;
Subject to: (5), (6), (7), (8).

Similarly, solving Problem 5 is as same as performing both the two levels of URA in the cell a, i.e. SCA and TPC. To guarantee the convergence of the URA game to a commonly agreed coexistence pattern, the adaptation of price factor $\delta^{(n)}$ in each $C^{(n)}_l$ is necessary to eliminate the interest conflicts among greedy cells in the game. We do not assume perfect global knowledge and centralized decision maker like virtual referee [17]. Intuitively, it is difficult to ensure that the non-cooperative URA game defined by $C^{(a)}_l$ always converges to the so-called globally optimal coexistence pattern created by the cooperative URA game defined by $C^{(a)}_g$. Moreover, even such coexistence pattern can be established via perfect inter-cell coordination, it can be subject to frequent change due to dynamic white-space environment and potential user mobility. Combined with Algorithm 1, a heuristic yet distributed Algorithm 2 is proposed as follows, which is able to create near-optimal coexistence patterns with low complexity.

Algorithm 2 Two-level game-theoretic algorithm

1: **set** $\delta^{(a)} = \delta^{(a)}_0$ ($\delta^{(a)}_0$ is a positive constant), and begin to participate in the SCA game
2: **loop**
3: **set** the backoff timer for a random time interval
4: **repeat**
5: **do** the countdown of backoff timer
6: **until** the backoff timer expires
7: **measure** $\tilde{I}^{(a)}_k$ for $k \in \mathcal{K}$, and compute $\tilde{L}^{(a)}$ under the current $\mathcal{U}^N$ and $\tilde{\mathcal{P}}^N$ generated in the TPC game
8: **set** $L_0 = \tilde{L}^{(a)}$, $I_0 = \sum_{k=1}^{K} \tilde{I}^{(a)}_k$, and $F_0 = L_0 + \delta^{(a)}_0 I_0$, and record $\mathcal{U}^{(a)}_0 = \mathcal{U}^{(a)}$
9: **run** Algorithm 1
10: **set** $L = \tilde{L}^{(a)}$, $I = \sum_{k=1}^{K} \tilde{I}^{(a)}_k$, and $F = L + \delta^{(a)} I$ under the updated $\mathcal{U}^N$ and $\tilde{\mathcal{P}}^N$ from Algorithm 1
11: **if** $F \geq F_0$ **then**
12: **set** $\mathcal{U}^{(a)} = \mathcal{U}^{(a)}_0$ (give up acting in the SCA game)
13: **end if**
14: **if** $!(L \leq L_0 \;\&\&\; I \leq I_0)$ **then**
15: **update** $\delta^{(a)} = \delta^{(a)} + \Delta$ (Δ is a positive constant)
16: **end if**
17: **end loop**

In Algorithm 2, the loop between lines 2 and 17 describes the cell a's behavior in the SCA game, which is on the first level of the URA game. At any time a new SCA strategy is chosen by the cell a, the TPC game as in lines 2 and 14 in Algorithm 1 is played and achieves the Nash equilibrium as in Lemma 3. The TPC game, on the second level of the URA game, is played more frequently than the SCA one. The use of backoff timer between lines 3 and 6 aims to simplify the scheduling of acting cells in the multi-cell system. The adaptation of $\delta^{(a)}$ in line 15 is to make the cell a contribute to the Nash equilibrium in the game, which is proved below.

Theorem 2 In the non-cooperative URA game defined by Problems 4 and 5, the cells that run Algorithm 2 independently agree on a common coexistence pattern under the established Nash equilibrium without inter-cell coordination.

Proof As in Lemma 3, the TPC game defined by Problem 4 can always achieve the Nash equilibrium whenever any cell in the game takes another SCA strategy. Then, we verify that the cells running Algorithm 2 can agree on a common $\mathcal{U}^N_0 = \dot{\times}_{n \in \mathcal{N}} \mathcal{U}^{(n)}_0$ in the SCA game with no inter-cell coordination, and nobody has the intention of changing $\mathcal{U}^N_0$. According to the price adaptation, $\delta^{(a)}$ in the cell a is increased by Δ if the taken SCA strategy does not contribute to the desired Nash equilibrium. The cell a contributes to the Nash equilibrium only when $L \leq L_0$ and $I \leq I_0$. If $\delta^{(a)}$ has been increased to always make $F \geq F_0$ hold, the cell a no longer takes any action in the SCA game. The adaptation of $\delta^{(n)}$ for $n \neq a$ is similar. Once all the cells give up acting in the SCA game, a certain common $\mathcal{U}^N_0$ is agreed. The Nash equilibrium is then established by the TPC game under the fixed $\mathcal{U}_0^N$. In the end, a commonly agreed coexistence pattern is created. □

The proposed two-level game-theoretic approach as in Algorithm 2 is heuristic, and does not guarantee the resulting coexistence pattern to be unique and globally optimal. However, it does not rely on any inter-cell coordination and has low complexity. Our two-level approach is always based on actual local interference measurements and does not need any estimates of coexisting networks. For these reasons, the proposed USFR technique and associated algorithm are suitable for the self-coexistence of uncoordinated CR networks that operate in dynamic white-space spectrum environment.

VII. Simulation Results

In this section, real-time performance of USFR is evaluated. We focus on a certain self-coexistence scenario, as in Fig. 2, where $N = 7$ CR networks are forced to share a single TV white-space channel. Each CR network is centrally controlled by a BS (triangle mark), and direct inter-network coordination is not available. In each network cell n, $M^{(n)}$ uplink sessions are generated and randomly distributed. The coverage radius of each cell is 5 km, and the propagation gains are computed via log-distance path-loss model with exponent 3.2. But note that the only information known to each BS n is the interference measurements $I_k^{(n)}$. The other parameters are set as follows: $\gamma_{m_n}^{(n)} = 30$, $T_{m_n}^{(n)} = 1$, $\bar{Q}_{m_n,k}^{(n)} = 100$ mW, and N_0 is given by noise density -174 dBm/Hz and TV channel width 6 MHz. The coexistence patterns created by Algorithm 2 under different combinations of $M^{(n)}$ and K are analyzed. For ease of comparison, $M^{(n)} = m$ for $n \in \mathcal{N}$. We define the number of subchannels per session by $\kappa \triangleq \frac{K}{7m}$, which characterizes the degree of spectrum sharing. As κ becomes smaller, more sessions have to share subchannels. When $\kappa \geq 1$, any session in the system get $T_{m_n}^{(n)} = 1$ subchannel. Note that a session with $T_{m_n}^{(n)} > 1$ can be regarded as multiple sessions with $T_{m_n}^{(n)} = 1$.

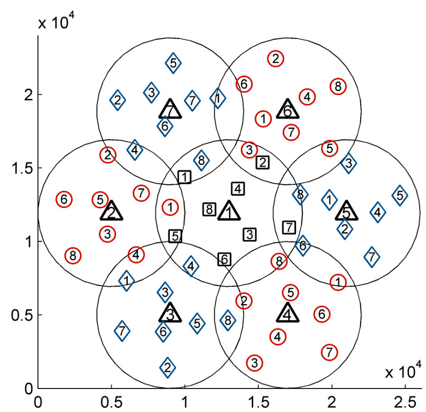

Fig. 2. A 7-cell simulation scenario.

First, we evaluate the convergence of USFR. The average number of iterations required to agree a common coexistence pattern is presented in Fig. 3. Here, one iteration represents a change of URA strategy by any cell in the 7-cell system. The starting point of $\mathcal{U}_0^N$ in Algorithm 2 is any feasible point to Problem 1. We can see that the convergence of USFR does not take too many iterations (per cell) even if the shared channel is crowded with mutually interfering sessions, i.e. small κ.

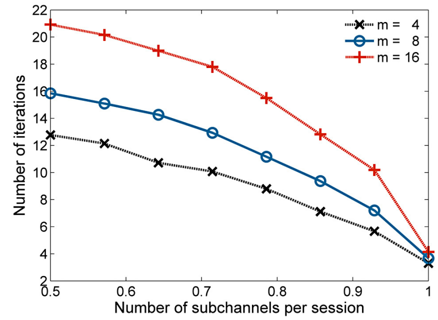

Fig. 3. Convergence of USFR ($\Delta = 1 \times 10^{10}$).

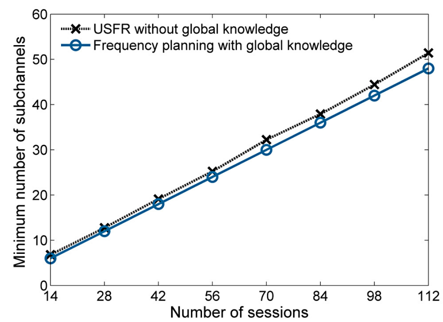

Fig. 4. Spectrum utilization of USFR.

Second, we evaluate the spectrum utilization of USFR. The distributed USFR is compared with the centralized frequency planning in Fig. 4 in terms of the minimum K required to accommodate $7m$ sessions. With the global knowledge of the 7-cell topology as in Fig. 2, the frequency planning typically requires $3m$ subchannels to accommodate $7m$ sessions with $T_{m_n}^{(n)} = 1$. This is because the cells 2, 4, and 6 can share the same subchannels safely, and so can the cells 3, 5, and 7. We can see that our USFR achieves a similar level of spectrum utilization without inter-cell coordination. In each cell, most inner users can be matched with edge users, probably far edge users, in other cells with acceptable mutual interference.

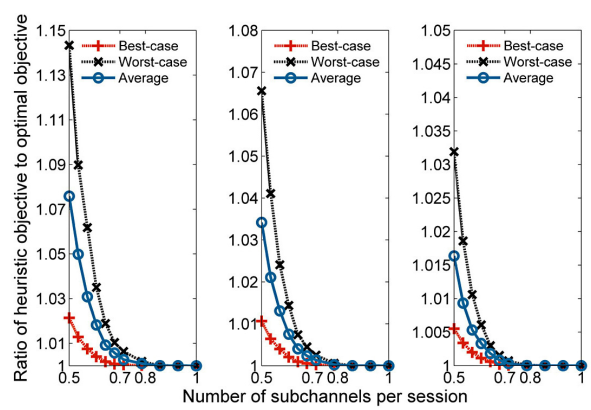

Fig. 5. Power consumption of USFR: (a) $m = 4$ (left); (b) $m = 8$ (middle); (c) $m = 16$ (right).

Third, we evaluate the power consumption of USFR. The objective ratio $\frac{\textit{heuristic}\text{ value of } G}{\textit{optimal}\text{ value of } G}$ is studied in Fig. 5, where the heuristic objective value is from the non-cooperative URA game under $C_l^{(n)}$, and the optimal one is from the cooperative URA game under $C_g^{(n)}$. Note that scheduling the cells in different orders may create different coexistence patterns, while in Algorithm 2, we just employ a random cell scheduling for coordination avoidance. Therefore, in addition to the average ratio value, we also compare the best-case and worst-case ones for a complete evaluation. We can see that our USFR achieves near-optimal performance in most cases, especially when the shared channel is not overcrowded, i.e. not too small κ.

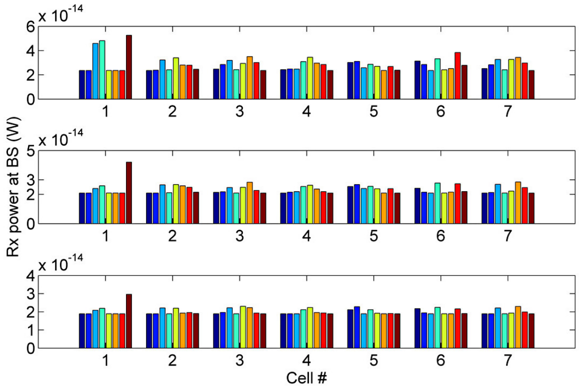

Fig. 6. A sample of sessions' received power levels at home BS: (a) $\kappa = \frac{4}{7}$ (top); (b) $\kappa = \frac{9}{14}$ (middle); (c) $\kappa = \frac{5}{7}$ (bottom).

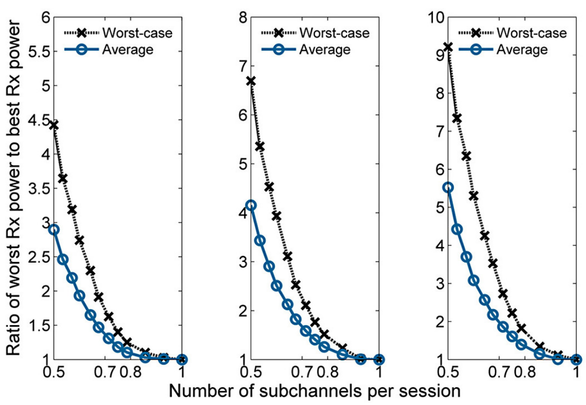

Fig. 7. Intra-cell fairness of USFR: (a) $m = 4$ (left); (b) $m = 8$ (middle); (c) $m = 16$ (right).

Finally, we evaluate the intra-cell fairness of USFR. We first focus on a specific scenario as in Fig. 2. A sample of sessions' received power levels at their home BS is shown in Fig. 6. As we can see, the received power levels at BS are generally same regardless the sessions' different link distances. The near edge users like sessions 1 and 2 in the cell 1 tend to take subchannels exclusively. But the inner users like sessions 3 and 4 in the cell 1 have to coexist with some users in other cells, and thus need even higher received power to guarantee SINR. Next, we focus on a general case. In each cell n, the fairness factor is defined by $\frac{\max_{m_n \in \mathcal{M}^{(n)}, k \in \mathcal{K}, U^{(n)}_{m_n,k} \neq 0} \{P^{(n)}_{m_n,k} H^{(n,n)}_{m_n}\}}{\min_{m_n \in \mathcal{M}^{(n)}, k \in \mathcal{K}, U^{(n)}_{m_n,k} \neq 0} \{P^{(n)}_{m_n,k} H^{(n,n)}_{m_n}\}}$, as investigated in Fig. 7. Besides the average ratio value over the cells, we also record the worst-case ratio among those of the 7 cells. We can see that our USFR always tries to improve fairness as more subchannels become available. The users wasting more power for path-loss save more power from interference control.

VIII. Conclusion

In this paper, the technique of USFR has been proposed in order to enhance the self-coexistence of CR networks operating in white-space spectrum. In view of the distributed nature of CR networks in a dynamic environment, we have resorted to the two-level game-theoretic approach that does not require direct inter-network coordination. In the non-cooperative game for multi-cell URA, the second-level TPC is solved optimally, while the first-level SCA is done heuristically without relying on global knowledge and high computational capability. Based on reasonable heuristic, our USFR has been shown to effectively improve self-coexistence jointly in spectrum utilization, power consumption, and intra-cell fairness.

References

[1] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications," *IEEE J. Sel. Area. Comm.*, Vol. 23, No. 2, Feb. 2005.

[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey," *Elsevier Comp. Netw.*, Vol. 50, No. 13, Sep. 2006.

[3] D. Gurney, G. Buchwald, L. Ecklund, S. Kuffner, and J. Grosspietsch, "Geo-Location Database Techniques for Incumbent Protection in the TV White Space," in *Proc. IEEE DySPAN'08*, Oct. 2008.

[4] Ofcom, "Digital Dividend: Geolocation for Cognitive Access," *Tech. Report from Ofcom*, Feb. 2010.

[5] IEEE 802.16 WG, "IEEE Standard for Local and Metropolitan Area Networks, Part 16: Air Interface for Broadband Wireless Access Systems, Amendment 2: Improved Coexistence Mechanisms for License-Exempt Operation," *IEEE Std.*, Jul. 2010.

[6] IEEE 802.22 WG, "IEEE Standard for Information Technology, Wireless Regional Area Networks (WRAN), Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands," *IEEE Std.*, Jul. 2011.

[7] Huawei, "Soft Frequency Reuse Scheme for UTRAN LTE," 3GPP R1-050507, *Tech. Report from Huawei*, May 2005.

[8] WiMAX Forum, "Mobile WiMAX - Part I: A Technical Overview and Performance Evaluation," *Tech. Report from WiMAX Forum*, Aug. 2006.

[9] A. L. Stolyar, and H. Viswanathan, "Self-Organizing Dynamic Fractional Frequency Reuse in OFDMA Systems," in *Proc. IEEE INFOCOM'08*, Apr. 2008.

[10] A. L. Stolyar, and H. Viswanathan, "Self-Organizing Dynamic Fractional Frequency Reuse for Best-Effort Traffic through Distributed Inter-Cell Coordination," in *Proc. IEEE INFOCOM'09*, Apr. 2009.

[11] H. Zhang, Q. Yang, F. Gao, and K. S. Kwak, "Distributed Adaptive Subchannel and Power Allocation for Downlink OFDMA with Inter-Cell Interference Coordination," in *Proc. IEEE GLOBECOM'10*, Dec. 2010.

[12] V. Tawil, W. Caldwell, I. Reede, T. Kiernan, and C. R. Stevenson, "DRAFT 802.22a (Amendment) PAR," IEEE 802.22-09/0029r0, *IEEE Std. PAR*, Jan. 2009.

[13] B. Rengarajan, A. L. Stolyar, and H. Viswanathan, "Self-Organizing Dynamic Fractional Frequency Reuse on the Uplink of OFDMA Systems," in *Proc. CISS'10*, Mar. 2010.

[14] F. Wamser, D. Mittelstädt, and D. Staehle, "Soft Frequency Reuse in the Uplink of an OFDMA Network," in *Proc. IEEE VTC'10-Spring*, Jun. 2010.

[15] H. Kwon, and B. G. Lee, "Distributed Resource Allocation through Non-Cooperative Game Approach in Multi-Cell OFDMA Systems," in *Proc. IEEE ICC'06*, Jun. 2006.

[16] Z. Liang, Y. H. Chew, and C. C. Ko, "Decentralized Bit, Subcarrier and Power Allocation with Interference Avoidance in Multicell OFDMA Systems Using Game Theoretic Approach," in *Proc. IEEE MILCOM'08*, Nov. 2008.

[17] Z. Han, Z. Ji, and K. J. R. Liu, "Non-Cooperative Resource Competition Game by Virtual Referee in Multi-Cell OFDMA Networks," *IEEE J. Sel. Area. Comm.*, Vol. 25, No. 6, Aug. 2007.

[18] T. Zhang, C. Feng, G. Su, and L. Xiao, "Uplink Multi-Cell Noncooperative Power Allocation Game Alogrithm for OFDMA Cellular Networks," in *Proc. WiCOM'09*, Sep. 2009.

[19] S. Sengupta, R. Chandramouli, S. Brahma, and M. Chatterjee, "A Game Theoretic Framework for Distributed Self-Coexistence among IEEE 802.22 Networks," in *Proc. IEEE GLOBECOM'08*, Dec. 2008.

[20] V. Gardellin, S. K. Das, and L. Lenzini, "A Fully Distributed Game Theoretic Approach to Guarantee Self-Coexistence among WRANs," in *Proc. IEEE INFOCOM'10*, May 2010.

[21] A. B. MacKenzie, and L. A. DaSilva, *Game Theory for Wireless Engineers*, Morgan & Claypool Publishers, Mar. 2006.