Social Networks 30 (2008) 247–258

Contents lists available at ScienceDirect

Social Networks

journal homepage: www.elsevier.com/locate/socnet

A multiple indicator approach to blockmodeling signed networks

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ARTICLE INFO

Keywords:
Generalized blockmodeling
Signed networks
Structural balance theory

ABSTRACT

Regardless of whether the focus is on algebraic structures, elaborating role structures or the simple delineation of concrete social structures, generalized blockmodeling faces a pair of vulnerabilities. One is sensitivity to poor quality of the relational data and the other is a risk of over fitting blockmodels to the details of specific networks. Over fitting blockmodels can lead to multiple equally well fitting partitions where choices cannot be made between them on a principled basis. This paper presents a method of tackling these problems by viewing (when possible) observed social relations as multiple indicators of an underlying affect dimension. Quadratic assignment methods using matching coefficients, product moment correlations and Goodman and Kruskal's gamma are used to assess the appropriateness of using the sum of observed relations as input for applying generalized blockmodeling. Data for four groups are used to show the value of this approach within which multiple equally well fitting blockmodels for single relations are replaced by unique (or near-unique) partitions of the summed data. This strategy is located also within a broader problem of blockmodeling three-dimensional networks data and suggestions are made for future work.

Following the introduction of blockmodeling by Lorrain and White (1971), an explosion of work followed (Hummon and Carley, 1993) that: (i) explored its mathematical foundations (e.g. Schwartz, 1977; White and Reitz, 1983; Batagelj, 1997; Doreian et al., 2005); (ii) provided practical computer algorithms to perform blockmodeling (e.g. Breiger et al., 1975; Burt, 1976; Batagelj et al., 1992); (iii) explored detailed role structures (e.g. White et al., 1976); (iv) provided algebraic interpretations of role structures (e.g. Bonacich, 1979; Winship and Mandel, 1983; Mandel, 1983) and (v) provided many empirical applications of blockmodeling. Generalized blockmodeling was presented as a general framework permitting many different block types with new types of blockmodels and direct fitting of blockmodels to social relational data that included blockmodels for signed relations (Doreian et al., 2005).

Regardless of whether the focus is on algebraic structures, elaborating role structures or the simple delineation of concrete social structures, there are at least two vulnerabilities for this approach: (1) possible poor quality of the relational data analyzed and (2) the risk of over fitting a blockmodel to the details of a specific network. This paper presents one way of tackling both problems through the use of multiple indicators for signed social relations. Section 1 introduces the problem more fully. The methods and the data are described in Section 2. These methods are applied to the data and the results are interpreted in Section 3. The final section discusses the empirical results, presents some suggestions concerning blockmodeling both signed and unsigned social relations and three-dimensional blockmodeling.

1. Signed social relations and blockmodeling

When studying sociometric structure, the modal strategy uses positive social relations. Yet, many social relations carry both positive and negative affect and the dynamics of signed affect ties between people are different to those of positive relations alone. Fortunately, there are signed network data sets available to explore these issues. For signed relations, one focus – informed by the work of Cartwright and Harary (1956) and of Davis (1967) – is on delineating the group structure in the sense of identifying mutually hostile subgroups where each subgroup is held together by positive ties.

Structural balance theory has its origins[1] in the work of Heider (1946). Cartwright and Harary (1956) generalized structural balance theory in three ways by: (a) removing the distinction between signed social relations and signed unit formation relations to study, simply, signed relations; (b) moving from studying signed triples (in the minds of actors) to studying signed social networks (of actors) (Doreian, 2004); (c) providing a compelling statement of the global

[1] While there are many variants of 'consistency theories', for example, Newcomb (1961, 1968), Festinger (1957) and Osgood and Tannenbaum (1955), Heider's formulation is used here as the point of departure.

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doi:10.1016/j.socnet.2008.03.005

structure of a group based on the microdynamics of signed relations among people. A signed network (Doreian et al., 2005, p. 298) is an ordered pair (G, σ) for which the following hold:

(1) $G = (V,A)$ is a digraph, without loops, where V is a set of vertices and A is a set of arcs such that $A \subseteq V \times V$, the Cartesian product of V with itself and
(2) $\sigma: A \to \{p,n\}$ is a sign function. The arcs with sign p are positive and arcs with sign n are negative. If the relation is binary then an equivalent representation is $\sigma: A \to \{+1,-1\}$.

The second part can be extended to valued networks where the arcs have both sign and magnitude. A finite sequences of vertices and arcs, $s = v_0, a_1, v_1, a_2, v_2, \ldots, a_m, v_m$ is a walk from v_0 to v_m in G if and only if for each value of i $(1 \le i \le m)$, a_i is (v_{i-1}, v_i), an arc from v_{i-1} to v_i. If s satisfies a weaker condition that every a_i is either (v_{i-1}, v_i) or (v_i, v_{i-1}) then s is a semi-walk. If $v_0 = v_m$, then s is a closed semi-walk. A semi-walk is positive if the product of the signs of its arcs is positive and a signed network is balanced if every closed semi-walk is positive. The basic theorem of Cartwright and Harary (1956) states: a signed network is balanced if and only if the set of vertices can be partitioned into two subsets such that every positive arc[2] joins vertices within the same subset and all negative arcs join vertices of different subsets. Davis (1967) proposed labeling the clusters as plus-sets (their only internal sign is positive) and established a generalization of Cartwright and Harary's result: a signed network is exactly partitionable into two or more plus-sets if and only if it has no closed semi-walks with exactly one negative arc.

Doreian and Mrvar's (1996) partitioning method, for signed networks, identify the partitions of these 'structure theorems'. More importantly, it can be used to delineate empirical partitions as close to these ideal forms as possible given empirical signed networks that are neither balanced nor exactly partitionable. Their method was located later within the generalized blockmodeling framework by defining new block types and a new blockmodel for signed networks, together with a criterion function (see below) to be minimized across all possible partitions to establish partitions that best fit the data directly (Doreian et al., 2005, Chapter 10).

The form of a blockmodel image, based on these structure theorems is simple: blocks on the main diagonal have only 1 s or 0 s while, off the main diagonal, all the block elements are either -1 or 0. Put differently, only 'positive blocks' appear on the diagonal and only 'negative blocks' are off the diagonal. This can be extended to include positive values and negative values for valued signed networks. The criterion function is based on the line indexes defined by Harary et al. (1965, pp. 348–351). One is the smallest number of lines for which a sign reversal leads to a balanced network. Another is the smallest number of lines whose removal leads to a balanced network. Harary et al. (1965) proved they are identical for all signed networks. Letting $\mathcal{P}$ be the number of positive ties where they should not be (between plus-sets), $\mathcal{N}$ the number of negative ties where they should not be (within plus-sets) and C a partition into k clusters, a simple criterion function, $\Phi(C)$, is shown in Eq. (1) (where $0 \le \alpha \le 1$). The parameter α allows an analyst to differentially weight the two types of inconsistencies when one type is thought to be more consequential than the other. Usually, however, α is set to 0.5 to weight $\mathcal{P}$ and $\mathcal{N}$ equally.

$$\Phi(C) = \alpha\mathcal{P} + (1-\alpha)\mathcal{N} \tag{1}$$

Doreian and Mrvar's (1996) algorithm is a local optimization procedure[3] that minimizes $\Phi(C)$ to get the partition(s) closest to a balanced partition (for which $\Phi(C)$ would be 0). For binary networks, $\mathcal{P}$ and $\mathcal{N}$ are counts of the number of arcs where they should not be according to balance. For valued signed networks they are the sums of the values of the ties that are inconsistent with structural balance.

Some of the early work on blockmodeling used multiple relations by 'stacking' them, computing measures of (dis)similarity across them and using clustering algorithms. When signed data were used by Breiger et al. (1975) and White et al. (1976), using the indirect approach, the positive and negative ties were separated into different matrices that were then stacked. Doreian et al. (2005, pp. 312–317) used signed network data from Bales (1970) and partitioned them in two ways: (i) using the above algorithm with the criterion function in Eq. (1) and (ii) separating the positive and negative ties into two relations, stacking them and then partitioning according to structural equivalence. The results were dramatically different and favored treating the signed ties together rather than treating them as separate, stacked, relations.

Doreian et al. (2005) focused on developing their generalized blockmodeling approach and most of their examples featured single relations. Marsden (2006) noted this and observed the earlier blockmodels used multi-relational data. The obvious inference is that the generalized blockmodeling approach can be – indeed, must be – extended to consider multiple relations. Yet how to do this is less than obvious, especially when the relations are different, for example, organizational hierarchies and informal social relations. Further, stacking signed networks to use indirect methods is not appropriate.[4] One response is to pursue three-dimensional blockmodeling of multiple relations (Baker, 1986; Batagelj et al., 2007). In cases where the multiple relations can be viewed as indicators of some underlying relational dimension, a multiple indicator approach could prove useful. This is adopted here for signed relations to deal with the potentially serious pair of problems outlined above when blockmodeling is used for one social relation.

Doreian et al. (2005) note that for any social relation, their methods could lead to multiple equally well fitting partitions. This can be viewed as interesting substantively (their view) or as a problem created by sensitivity to measurement errors in one social relation. More precisely, having multiple equally well fitting partitions may be a result of measurement errors in the indicators when they are used as measured relations rather than as indicators. When there are equally well fitting partitions – partitions with the same minimized value of $\Phi(C)$ for one or more values of k – these partitions cannot be distinguished in terms of fit. The companion problem is that blockmodels of a single relation run the risk of being over fitted. Over fitting models of any sort – be they formal, empirically estimated equations or even rich ethnographic descriptions – is a major problem for the social sciences (de Marchi, 2005). A multiple indicator approach to blockmodeling, as proposed here, can help solve these problems by identifying unique best fitting partitions and not having blockmodels over fitted to idiosyncratic features of a *particular* relation.

[2] If the signed network contains both arcs (directed lines) and edges (undirected lines) then the structure theorems can be restated using lines instead of arcs.

[3] A network is partitioned randomly into k clusters and the criterion function is computed. The neighborhood of a clustering is defined by two types of transitions: (a) moving a vertex from one cluster to another and (b) interchanging two vertices between clusters. If a transition leads to a lower value of the criterion function, the new partition is chosen and the procedure is repeated until no further improvement is possible. This is repeated many times to avoid a local minimum for the criterion function.

[4] There is no (dis)similarity measure for the extent to which actors belong together in plus-sets, and how the network ties are distributed into positive or negative blocks, which rules out the usual stacking for signed networks.

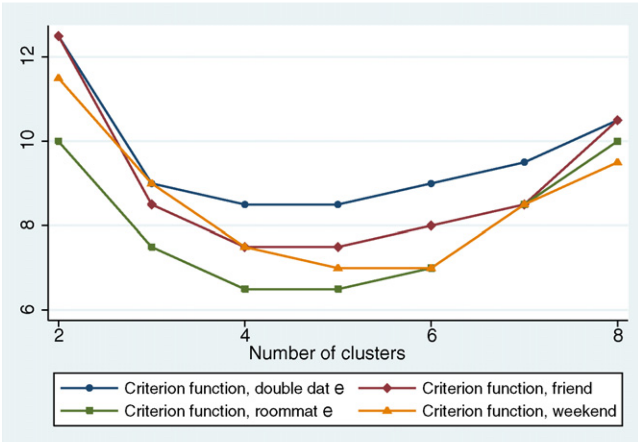

Fig. 1. Plot of the optimal values of the criterion function for each relation: House A.

2. Data and methods

Data used here come from Lemann and Solomon (1952) and Sampson (1968). Lemann and Solomon (1952) collected data from female students living in three "off campus dormitories" at an Eastern college. These residences were labeled Houses A, B and C, a usage continued here. They write (1952, p. 14) "sociometric criteria refer to different areas of choice" and, wanting to measure signed relations adequately, they add "multiple criteria broaden the base for acceptance or rejection." In their view, multiple criteria make it more likely that signed relations are measured adequately. This is consistent with measuring of an underlying affect dimension with multiple indicators. Four indicators were created by asking the women to name three others with whom they would like to do something and three others with whom that person would not like to do these things.[5] The activities were going on a double date, wanting another woman as a friend after college, having someone as a roommate and taking someone for a weekend visit with family.

These data can be examined in two ways: (i) analyzing each relation separately and (ii) using a multiple indicator approach for measuring an underlying affect dimension. To consider this we need the following result (Doreian et al., 2005, pp. 305–306) regarding the values of the criterion function in Eq. (1), where k is the number of plus-sets in a partition ($2 \le k \le n$), and partitions with k and ($k-1$) plus-sets are adjacent:

Theorem 1. For any signed network, (G, α), there is a unique lowest value of the criterion function. This value will occur for partitions with a single number of (one value of k) plus-sets or for adjacent partitions (and more than one value of k).

The basic problem of delineating multiple partitions is described in Fig. 1 (for House A) and in Table 1 (for all houses). They demonstrate the realization of the potential problem when the four relations for House A are analyzed separately. The curve of the criterion function plotted against k is concave upwards, consistent with Theorem 1, and is shown in Fig. 1 for each relation fitted separately in House A. (Given Theorem 1, it is not necessary to establish partitions for every possible value of k.) For double date, friend after college and preferred choice for a roommate, the minimum values occur for $k = 4$ and $k = 5$. The minimum value of $\Phi(C)$ *for* the weekend visit relation occurs for $k = 5$ and $k = 6$. This ambiguity is bad enough (even though a case can be made for choosing $k = 5$ as a compromise). A deeper problem revealed in Table 1 is the presence of multiple equally well fitting partitions for all of the relations. The columns list the relations, the values of k for which $\Phi(C)$ is minimized, the optimized values of $\Phi(C)$, one for each value of k, and the number of optimal partitions are in the last column. Within each house, there is no unique optimal partition across all relations. Moreover, there is no agreement over the best k across the four relations. Both features are undesirable. The second two panels of Table 1 show that the problem of multiple equally well fitting partitions applies to the other houses, albeit in a less acute form for House B for two of the four relations.

Table 1
Details on the ambiguity of separate partitions of the observed relations

Signed relation	k	Optimal criterion function	Number of optimal partitions
House A			
Double date	4	8.5	2
	5	8.5	2
Friend after college	4	7.5	1
	5	7.5	1
Roommate	4	6.5	4
	5	6.5	3
Weekend visit	5	7.0	5
	6	7.0	5
House B			
Double date	4	9.0	2
Friend after college	4	9.0	1
Roommate	4	8.5	1
Weekend visit	3	9.5	1
	4	9.5	9
House C			
Double date	3	6.5	3
	4	6.5	1
Friend after college	3	7.0	3
	4	7.0	2
Roommate	3	9.0	1
	4	9.0	2
	5	9.0	1
Weekend visit	3	7.0	3

When viewing the measured ties as indicators of an underlying signed affect tie, there are options for proceeding. One simple tactic is to sum the four relations to get a valued signed network. Moreover, the values of the summed relation have a simple interpretation. If v_{ij} is the summed value of the separate ij ties, across all relations, it is a valued tie from actor i to actor j. If $v_{ij} = v$ then actor i has chosen actor j for v of the four possible activities. Similarly, if $v_{ij} = -v$ then i has reported that she would not like doing v activities with j. The range of ties values is given by $-4 \le v_{ij} \le 4$ and the values form a ratio scale over this range.

Fig. 2 provides a plot of the optimized value of $\Phi(C)$ for House A against k for the summed relation. There is a unique value of k for the minimized criterion function this partition is unique (for $k = 4$). Using multiple indicators permitted a unique optimal partition for House A.

There are two potential complications to consider. One occurs if an individual, i, wants to do some activities with j and wants to not do other activities with j. The value of the summed tie, v_{ij}, is then the net number of activities that i wants to do with j and can be interpreted as such. But if there are many such instances then the utility of the multiple indicator approach suggested here may be compromised. Fortunately, this never happens for House A and happens only once (over 162 non-zero choices) in House B. For House C this happens 6 times out of the 208 non-zero choices. The

[5] A debate on the appropriateness (or not) of asking for a fixed number of choices was triggered by Holland and Leinhardt (1973). While fixed choice instruments are still used, the consensus now appears to be that it is better to have a free number of nominated others in a sociometric instrument. However, Lemann and Solomon (1952) provide a quite sophisticated argument for their use of a fixed choice design.

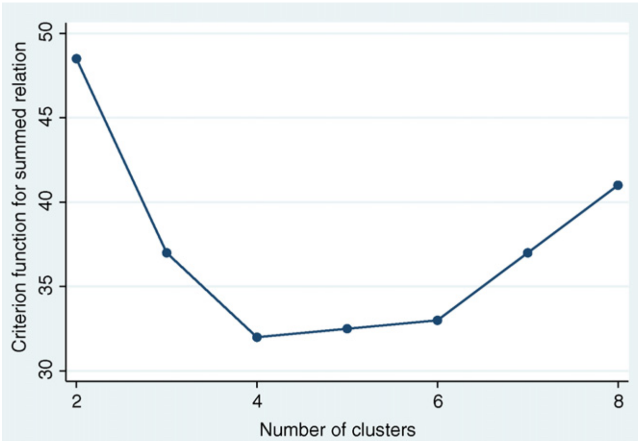

Fig. 2. Optimal values of the criterion function for the summed relation: House A.

incidence of these conflicting choices is minimal to non-existent in the House data.

A second potential problem is that the multiple indicators might not 'scale' in the sense that the observed relations are not indicators of an underlying dimension. Put differently, it is necessary to see if the 1 s and −1 s appear generally in the same places in the multiple 'observed' relations. To check this, a direct approach was taken by using quadratic assignment procedures (QAP), as implemented in Ucinet (Borgatti et al., 2002) with a focus on the simple matching coefficient (Sokal and Michener, 1958), Goodman and Kruskal's (1954) gamma and also product moment correlations. The QAP results are robust in the face of interdependence (Krackhardt, 1987). Given the significance of the correlations, a secondary check was done using the first principal component of the observed indicators, constructing the underlying factor, and examining the loadings for consistency.[6] The summed variable and the constructed variable (from a principal components analysis) ought to be close to each other if the overall strategy has merit. When this criterion or constraint is met, it makes sense to use generalized blockmodeling with the summed relation alone rather than use it for each relation separately.

3. Applications

The above strategy is applied first for the House data to provide separate assessments of the multiple indicator approach. These analyses were performed using Ucinet and Stata (StataCorp., 2003). With adequacy of the underlying single model established, the generalized blockmodeling results for the summed relation are then presented.

3.1. *Assessing the appropriateness of a single factor: House data*

Table 2 reports the simple matching coefficients for the positive and negative ties (separately) in one panel. The second panel of this table has the correlation coefficients for relations in their signed form together with the values of Goodman and Kruskal's gamma. The QAP generated *p*-value or *every* coefficient reported in this table is 0.000 (using 2500 permutations) and is not reported in the table. In general, the matching coefficients for the positive ties are slightly above those for the negative ties. However, all are large as well as significant. Similarly, the values of the product moment correlations between the observed relations, based on an assumed linear relation, are high with higher correlations with the summed relation. The values of the Goodman and Kruskal gamma statistic, treating the observed ties as ordinal, are also very high. For Houses A and B, the value of this statistic is 1 for the relation between the summed relation and each of the observed relations.[7]

The results of a principal components factor analysis of the observed relations are reported in Table 3. For House A, the eigenvalues are 3.007, 0.425, 0.333 and 0.236. The corresponding values for House B are 3.114, 0.360, 0.277 and 0.249. Finally, the eigenvalues for House C are 2.824, 0.436, 0.394 and 0.346. For each house, the first far exceeds the remaining eigenvalues and the first factor accounts for 75%, 78% and 71% of the common variance for Houses A, B and C, respectively. The factor loadings and uniqueness values in Table 3 are uniform in each panel.[8] Using a single underlying variable in the form of the sum of the relations for generalized blockmodeling seems appropriate for these data.

3.2. *Generalized blockmodeling partitions: House data*

Generalized blockmodeling of signed relations delineates the macro structure of groups in terms of the number of plus-sets, the membership composition of these plus-sets and the level of imbalance in the group. Fitting these blockmodels requires the use of a local optimization procedure that is repeated many times (as described in footnote 4). This is implemented in Pajek (Batagelj and Mrvar, 1998). For the relevant values of *k* in each house, the optimization was repeated 2000 times. The summary of the fitting process is shown in Table 4. For each house, the following results are shown: the values of *k* for which optimal partitions were obtained (from 2 through 6, recall Theorem 1), the optimal value of $\Phi(C)$ for each *k* and the number of optimal partitions for each value of *k*. The overall optimal values are bolded. For House A, there is a unique partition of the women into 4 plus-sets when $\Phi(C)$ is 32 (the measure of imbalance). For House B, the optimal value of $\Phi(C)$ (40.5) is found also for partitions into four plus-sets. However, there are two equally well fitting partitions. There is a unique optimal partition also for House C, with $\Phi(C)$ taking the value of 31.5, with 3 plus-sets in the optimal partition.

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[6] Using Cronbach's (1951) Alpha, at face value, is another way of examining the consistency of the observed indicators, in the sense of seeing if they tap a single underlying dimension. However, in computing this measure, using the $(n(n-1)/2$ dyadic elements (for n members of a house) of the sociomatrices implies severe non-independence of the observations together with the presence of many (0, 0) data points and makes this measure problematic.

[7] The cross-tabulation of the summed relation for house A and the double date relation is

Summed relation	Double date			Total
	−1	0	1	
−4	20	0	0	20
−3	13	10	0	23
−2	14	13	0	27
−1	16	33	0	49
0	0	209	0	209
1	0	12	11	23
2	0	9	7	16
3	0	8	7	15
4	0	0	38	38
Total	63	294	63	420

with no inversions of order between the two variables, the Goodman Kruskal gamma measure is at its maximum value of 1. This holds for all relations in Houses A and B. House C has small numbers of inversions and these coefficients are slightly below 1.

[8] Further, tests of a single factor model against the saturated model, show the single factor model is statistically indistinguishable from the saturated model in all houses.

Table 2
Measures of correspondence for the observed relations: House data

	Positive ties			Negative ties		
	D	F	R	D	F	R
Matching coefficients: positive ties and negative ties						
House A						
Friend	0.929	1.0		0.857	1.0	
Roommate	0.900	0.929	1.0	0.867	0.886	1.0
Weekend visit	0.914	0.962	0.943	0.852	0.895	0.871
House B						
Friend	0.904	1.0		0.890	1.0	
Roommate	0.860	0.912	1.0	0.882	0.860	1.0
Weekend visit	0.868	0.926	0.926	0.912	0.882	0.868
House C						
Friend	0.905	1.0		0.853	1.0	
Roommate	0.905	0.911	1.0	0.853	0.853	1.0
Weekend visit	0.889	0.921	0.905	0.874	0.863	0.874

	Correlations				Goodman and Kruskal's gamma			
	D	F	R	W	D	F	R	W
Product moment correlations and values of Goodman and Kruskal's gamma								
House A								
Friend	0.643	1.0			0.818	1.0		
Roommate	0.611	0.690	1.0		0.774	0.872	1.0	
Weekend visit	0.611	0.762	0.690	1.0	0.774	0.931	0.872	1.0
Relation sum	0.827	0.893	0.863	0.884	1.0	1.0	1.0	1.0
House B								
Friend	0.725	1.0			0.861	1.0		
Roommate	0.647	0.686	1.0		0.778	0.836	1.0	
Weekend visit	0.706	0.745	0.716	1.0	0.836	0.884	0.873	1.0
Relation sum	0.872	0.895	0.864	0.897	1.0	1.0	1.0	1.0
House C								
Friend	0.571	1.0			0.822	1.0		
Roommate	0.588	0.605	1.0		0.799	0.799	1.0	
Weekend visit	0.605	0.647	0.630	1.0	0.799	0.832	0.832	1.0
Relation sum	0.823	0.840	0.840	0.858	0.985	0.985	0.993	0.993

While the problem of obtaining partitions multiple equally well fitting optimal partitions was apparent for single relations, using the sum of these relations solved the problem for Houses A and C while coming very close for House B. The two House B partitions are very close so that the same macro structure is described in both partitions.

3.2.1. *House A*

Fig. 3 shows the unique optimal partition into four clusters (plus-sets). The valued entries in the table vary from +4 to −4. Positive ties are represented by squares and negative ties are represented by diamonds. For both, values of the ties are represented by differential darkness of the shading with the darkest squares representing +4 and the darkest diamonds are used for −4. The theoretically driven blockmodel for signed networks posits positive blocks on the diagonal and negative blocks off the diagonal. The summary blockmodel is

Positive	**Negative**	**Negative**	**Negative**
Mixed	Mixed	Negative	Negative
Negative	**Negative**	**Positive**	**Null**
Mixed	**Negative**	**Negative**	**Positive**

where the blocks are labeled positive, negative, mixed and null.[9] The positive blocks are bolded when they are consistent with the specified blockmodel either by having no negative elements or very few (one or two) of them. Blocks where the (inconsistent) negative elements approach the number of positive elements are labeled as mixed. Block labels are not bolded when there are additional negative elements (between three and the number of ties identifying the block type). There is one exact null block. Negative blocks have been treated in a similar fashion: they are bolded when there are zero or very few positive elements in them through to being labeled as 'mixed' when the number of (inconsistent) positive elements approaches the number of negative elements. The label is used simply to indicate concern that the partition, while being the

Table 3
Principal components factor analysis for the multiple indicator model

Signed social relation	Loadings	Uniqueness
House A		
Double date	0.8199	0.3277
Friend after college	0.8962	0.1967
Room mate	0.8631	0.2550
Weekend visit	0.8868	0.2135
House B		
Double date	0.8716	0.2403
Friend after college	0.8960	0.1973
Room mate	0.8621	0.2568
Weekend visit	0.8988	0.1921
House C		
Double date	0.8196	0.3283
Friend after college	0.8405	0.2935
Room mate	0.8403	0.2940
Weekend visit	0.8603	0.2600

[9] Normally, the blocks are labeled 'positive' or 'negative', consistent with structural balance with the value of the criterion function noted. The additional labeling is simply a step in the direction of looking at the inconsistencies more closely.

Table 4
Summary of structural balance partitions for the three houses

House A			House B			House C		
k	Optimal criterion function	Number of partitions	*k*	Optimal criterion function	Number of partitions	*k*	Optimal criterion function	Number of partitions
2	48.5	1	2	51.5	2	2	34.0	1
3	37.0	1	3	42.0	1	3	**31.5**	1
4	**32.0**	1	4	**40.5**	2	4	33.0	1
5	32.5	2	5	41.5	1	5	33.0	1
6	33.0	1	6	43.5	1	6	37.5	1

best that can be done with pure structural balance, may point to additional mechanisms generating signed ties.

The 21 women are partitioned into 4 four plus-sets for the smallest value of the criterion function: p-s_1 = {g, h, o, u}; p-s_2 = {l, n, t}; p-s_3 = {a, b, d, e, f, i, j, k, m, p, q} and p-s_4 = {c, r, s}. Of these plus-sets, p-s_1 is the most consistent with structural balance ideas. All of the positive choices, at the maximum possible value of 4, made by g, h, o and u are within their plus-set. All of their negative choices are to women in two of the other plus-sets. The women of p-s_1 recognize none of the women in p-s_2 leading to the null block in Fig. 3. Also, they receive only two positive ties from women in other plus-sets. The plus-set, p-s_2, is consistent with balance having only internal positive ties. Given its size (of 3) and the number of choices made, some positive ties must go to women in other plus-sets. All of their negative ties go to women in other plus-sets. As for p-s_1, the women in p-s_2 receive only two positive ties from women in other plus-sets.

The largest plus-set is p-s_3 with 11 members. All but 3 of their 33 positive valued ties are to other members within their plus-set, consistent with structural balance. However, there are 10 negative ties within the plus-set. Again, most of their negative ties are sent to members of other plus-sets. They do receive positive ties from women in two other plus-sets, p-s_1 and p-s_4. The small plus-set, p-s_4, is the least consistent with structural balance. There are almost as many negative ties within the plus-set as there are positive ties. The members of p-s_4 send positive ties to members of all the other three plus-sets. However, its members receive only two positive ties from women in p-s_3 and none from women in the remaining plus-sets. The balance theoretic partition is clear and the departures are interpretable.

Measures of imbalance, whether constructed from cycles (Hummon and Fararo, 1995) or line counts (Harary et al. (1965) are computed for the network as a whole. While the line index is useful, because it provides the basis for the criterion function used, the above description of the balance partition of House A makes it clear that *balance holds differentially across the whole network*. In addition to knowing the composition of the plus-sets, this is an important insight masked by the use of the imbalance measures

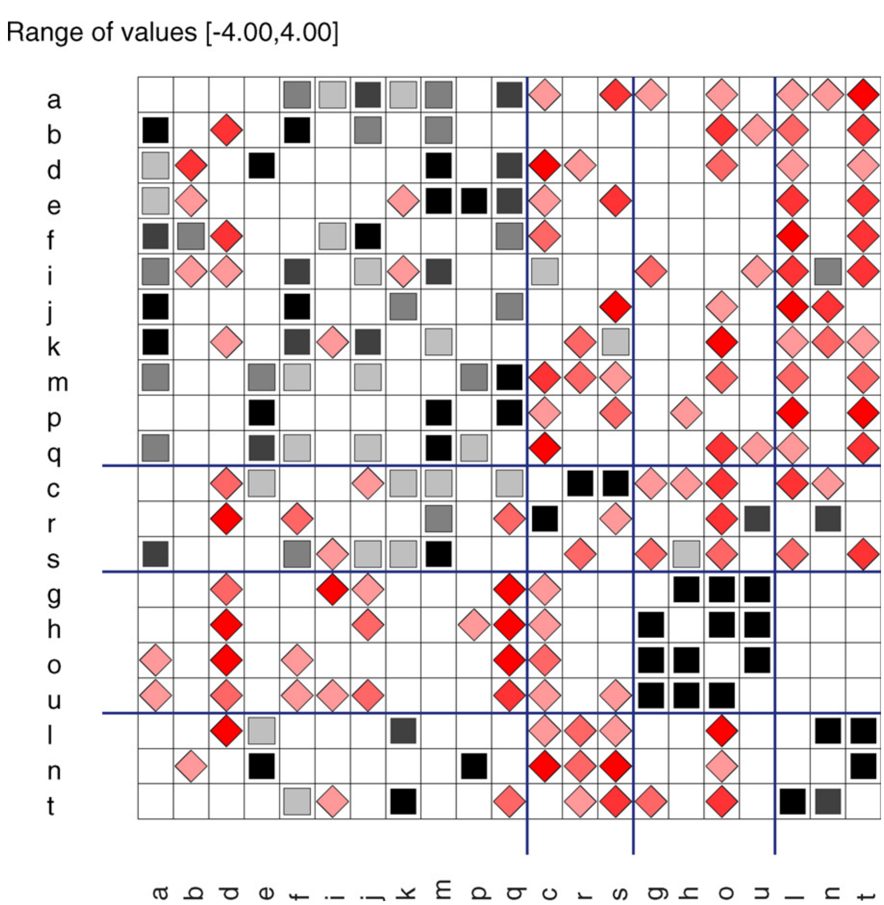

Fig. 3. Unique partition of the summed relations for House A.

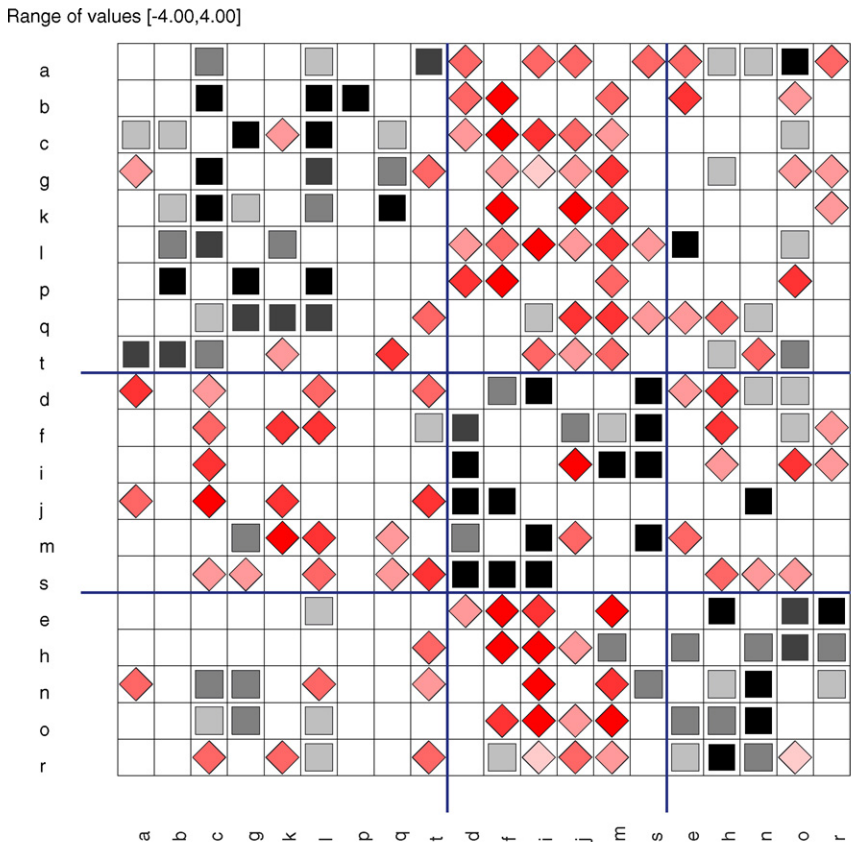

Fig. 4. Unique partition of the summed relations for House C.

alone in characterizing groups. Having balance holding to different extents across the network is part of the identified macro-structure of the group.

3.2.2. House C

The unique optimal partition for House C, in terms of structure balance, is shown in Fig. 4 and the structure of the blockmodel is

Positive	**Negative**	Mixed
Negative	**Positive**	Negative
Mixed	Negative	**Positive**

The three plus-sets are p-s_1 = {a, b, c, g, k, l, p, q, t}; p-s_2 = {d, f, I, j, m, s} and p-s_3 = {e, h, n, o, r}. The three plus-sets appear to be roughly equally consistent with balance theory regarding the absence of negative ties between the members within a plus-set. The number of these inconsistencies (6, 3 and 1, respectively) varies directly with the size (9, 6 and 5, respectively) of the plus-sets. The ties between the plus-sets p-s_1 and p-s_2 conform largely to structural balance with just one positive tie between the two sets of members. This is less true for p-s_1 and p-s_3 where there are fewer negative ties between members of the different plus sets and more positive ties between these two plus-sets. The members of both p-s_2 and p-s_3 send positive ties to each—but they are far fewer than the number of negative ties between the two plus-sets. Overall, they are the least consistent with balance on this criterion and there are plenty of null ties in the corresponding blocks. Again, there are differences with regard to balance holding across different parts of the structure albeit in a different form compared to House A.

3.2.3. House B

For seeking a unique optimal partition of the underlying signed relation, the approach proposed here is less successful although there are only two equally well fitting optimal partitions. One of these is shown in Fig. 5 for which the summary blockmodel is

Positive	**Negative**	**Negative**	Negative
Mixed	**Positive**	Negative	Mixed
Mixed	**Negative**	**Positive**	Mixed
Mixed	**Negative**	**Negative**	**Positive**

As was the case for House A, there are 4 plus-sets (with one of them much larger that the other three plus-sets). They are p-s_1 = {g, n, q}, p-s_2 = {i, j}, p-s_3 = {c, o} and p-s_4 = {a, b, d, e, h, f, k, l, m, p}. The only difference between this partition and a second equally well fitting partition is that i and f change places in their plus-sets. These partitions are very close to each other and qualitative interpretations of the two are identical.

The three small plus-sets all have only positive ties within them. However, their small size, especially acute for p-s_2 and p-s_3, together with the fixed choice design of Lemann and Solomon, means that at least half of the positive choices for women in these plus-sets must go to women in other plus-sets. Departures from balance theory are inevitable. The large plus-set, p-s_4, has many more positive (and stronger) ties than negative ties among its members. The block of ties from p-s_4 to p-s_3 is very close to perfect consistency with balance by having only a single positive tie (from b to c) among the many negative ties in the corresponding negative block. There are only two positive ties between p-s_4 and p-s_2 – with fewer negative ties – and there are only three positive ties from p-s_4 to p-s_1 with many negative ties in this block. The main departures for balance are due to the members of the small plus-sets sending their positive ties to members of the other plus-sets, especially to p-s_4, the largest plus-set. The use of a summed relational variable constructed from multiple indicators when used for partitioning networks, based on structural balance, has led to unique optimal

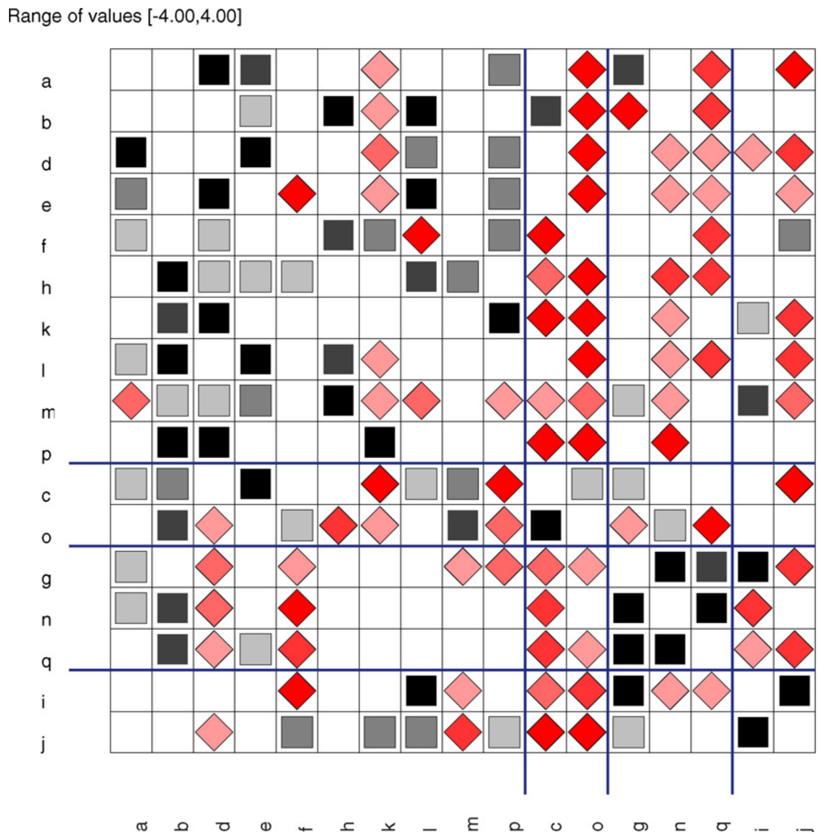

Fig. 5. One of two optimal partitions of the summed relations for House B.

Table 5
Measures of correspondence for the observed relations: Sampson T4

	Positive ties			Negative ties		
	Affect	Esteem	Influence	Affect	Esteem	Influence
Matching coefficients: positive ties and negative ties						
Esteem	0.843	1.0		0.843	1.0	
Influence	0.869	0.882	1.0	0.827	0.833	1.0
Sanction	0.706	0.729	0.729	0.801	0.794	0.791

	Correlations				Goodman and Kruskal's gamma			
	Affect	Esteem	Influence	Sanction	Affect	Esteem	Influence	Sanction
Product moment correlations and values of Goodman and Kruskal's gamma								
Esteem	0.706	1.0			0.846	1.0		
Influence	0.623	0.752	1.0		0.856	0.858	1.0	
Sanction	0.474	0.580	0.562	1.0	0.601	0.695	0.669	1.0
Relation sum (3)	0.869	0.920	0.887	–	0.986	0.995	0.993	–
Relation sum (4)	0.836	0.905	0.874	0.759	0.963	0.986	0.992	0.953

partitions for two of the three houses. For the third house there were two very close partitions of the actors and signed relation.

3.3. A second example: Sampson data

One of the most studied sociometric data sets comes from Sampson (1968), a usage triggered by the early blockmodeling analyses of Breiger et al. (1975) and White et al. (1976). Doreian and Mrvar (1996) returned to these data that were defined after part of a class graduated and a new group of trainees joined the monastic order for three time points (T2, T3 and T4). They focused on the single relation 'affect' and found the same unique partition for T2 and T3. They identified this partition also for T4 together with a second equally well fitting partition and remarked that examining closely the second partition could be interesting substantively. A rival interpretation is that the second partition is spurious and resulted from partitioning a single relation. The relations for which Sampson (1968) obtained signed data were 'affect', 'esteem', 'influence' and 'sanction'. (See Sampson (1968, pp. 316–318 and 457–472) for the operationalization of the variables and the data, respectively.) He measured them at the ordinal level. Even so, we sum these relations also.[10] Table 5 shows the results using the same matching coefficients and correlational measures used in Table 2. Again, the

[10] If we retained the operationalization of the summed relation as the count of the number of occurrences of signed ties, by binarized relational data first, we get the same partitions.

p-value for all coefficients in Table 5 is 0.000. The displayed values in the top panel suggest that sanction is the least consistent with the other indicators. And when the sanction relation is examined closely, three rows have only 0s for three trainee monks who did not sanction their brethren. The correlations for summed relation for three observed relations (affect, esteem and influence) are shown in the row labeled 'Relation Sum (3)' and the corresponding values for using all four relations are in the row labeled 'Relation Sum (4)'. These suggest that using only the three indicators is preferable. The results for the principal component analyses are shown in Table 6 and point to the same conclusion.

Fig. 6 shows the unique partition when the balance theoretic signed blockmodel is fitted to the summed data with the three positions of Young Turks, Outcasts and Loyal Opposition. The names of the trainee monks are grouped into clusters listed on the left. This partition is the same as reported by Doreian and Mrvar (1996) for T2 and T3 as well as one of the partitions they reported for T4. The partition structure of the blockmodel is:

Positive	**Negative**	Negative
Negative	**Positive**	**Negative**
Negative	**Negative**	**Positive**

Note that both White et al. (1976) and Doreian and Mrvar (1996) cite ethnographic detail in Sampson (1968) justifying the inclusion of Amand with the Outcasts. The decline in the value of the criterion function across the three time points follows the same pattern as described by Doreian and Mrvar (1996). The appealing feature of the multiple indicator approach is that it produces the same unique partition for all time points with a declining value of imbalance.

Table 6
Principal components factor analysis for the T4 monastery data

Signed social relation	Loadings	Uniqueness
All four indicators		
Affect	0.8293	0.3122
Esteem	0.8705	0.2423
Influence	0.8630	0.2552
Sanction	0.7390	0.4539
For the first three indicators		
Affect	0.8678	0.2470
Esteem	0.8897	0.2084
Influence	0.8714	0.2407

Therefore, there is little reason to worry about the second partition identified by Doreian and Mrvar for T4.

4. Discussion and suggestions for future work

Structural balance theory when coupled with generalized blockmodeling techniques provides an integrated, substantively based, approach for the analysis of signed data that leads to the delineation of the macro structure of human groups. Both the Lemann and Solomon (1952) and the Sampson (1968) data sets provide us with four measures of signed relational data. When these relations for the House data are analyzed separately there are multiple equally well fitting partitions across the different relations. Yet, when the multiple indicator approach is adopted, with relations viewed as observed measurements of some unobserved underlying signed

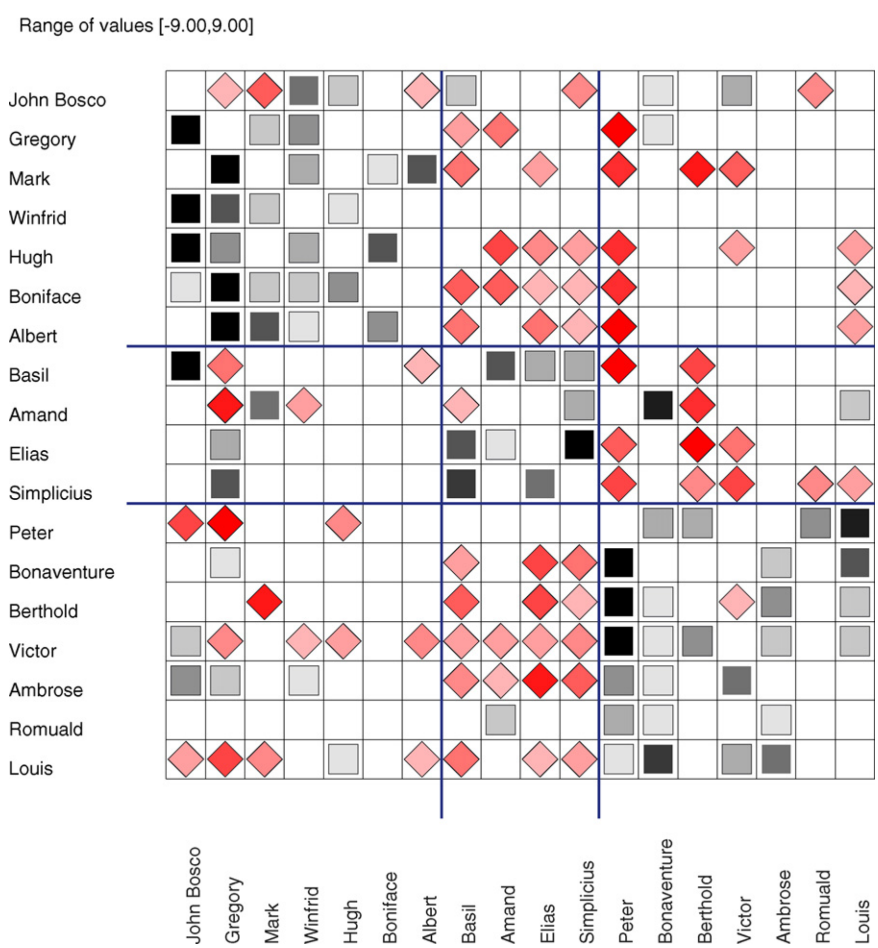

Fig. 6. Unique partition of the summed Sampson T4 data (three relations).

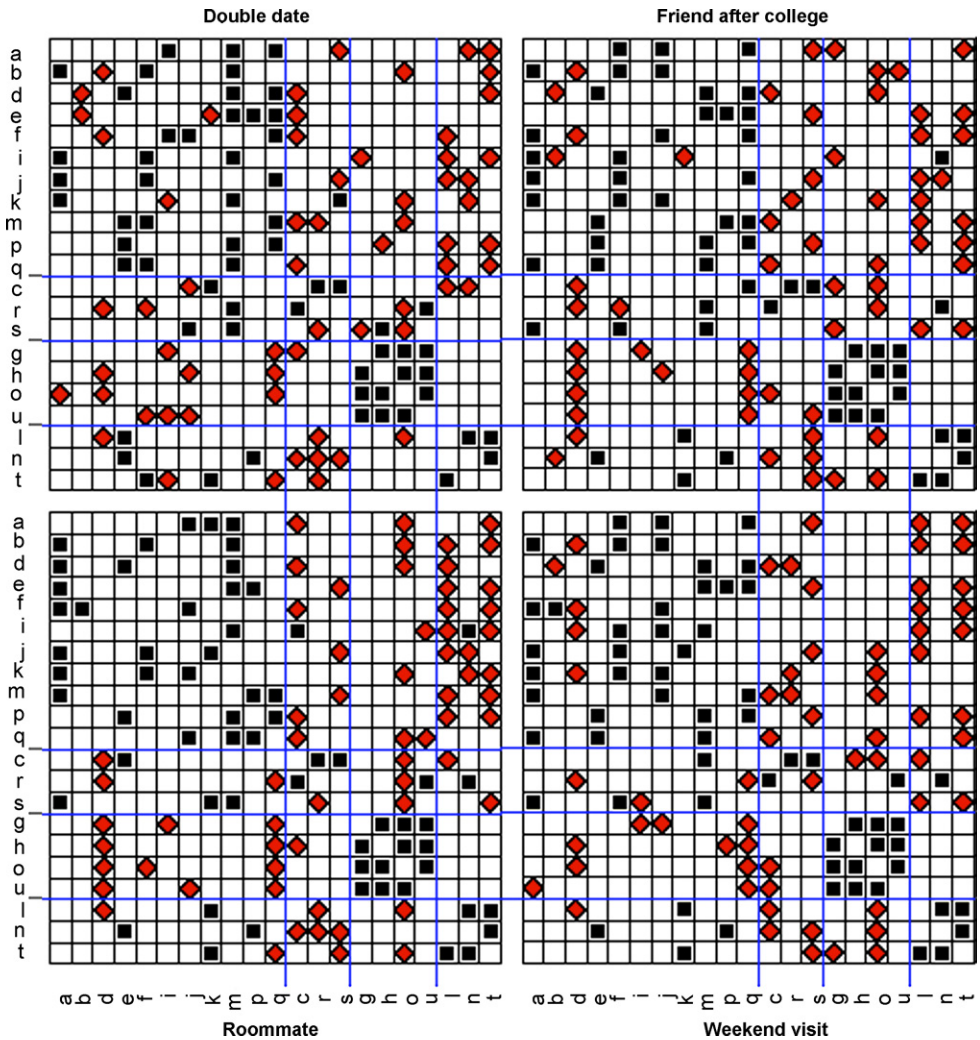

Fig. 7. The unique partition imposed on the observed relations for House A.

affect relation for the House data, the results are consistent and interpretable. For two of the Houses (A and C) the approach establishes a unique optimal partition. The analysis of the data for the third House (B) leads to two optimal partitions that are extremely close to each other and have the same qualitative interpretation. The problems with the Sampson data are far less acute as far as multiple partitions are concerned where fitting a signed blockmodel to the sum of three relations also produces a unique partition for T4.

The unique partition described above can be imposed on the separate relations for the House Data. This is shown in Fig. 7 for House A. (Similar results occur for the other houses and are not reported here.) Computing the measures of fit for each relation under the imposed partition reveals that these measures are, in general, slightly larger than the optimized value of the criterion function for each relation. They never take a lower value as shown in Table 7. In fitting blockmodels, in general, two views can be taken: (i) minimizing the criterion function is most important and (ii) obtaining a unique partition is preferable to having many equally well fitting partitions. These views hold for fitting a blockmodel to one relation or to multiple relations. If unique optimal partitions are identified, there is no contradiction between the two views. However, when they do not agree a choice has to be made. Stressing the first view suggests that the partitions with the optimized values of $\Phi(C)$ in the first column (in Table 7) are preferable while stressing the second view leads to accepting those in the second column. There are two arguments supporting the second view. One is that having a principled reason for selecting a partition is better

Table 7
Comparison of criterion function values for the House data

	Value of the optimized criterion function under over fitting	Value of the criterion function for the imposed partition
House A		
Double date	8.5	9.5
Friend after college	7.5	8.0
Roommate	6.5	6.5
Weekend visit	7.0	8.0
House B		
Double date	9.0	10.0
Friend after college	9.0	9.5
Roommate	8.5	10.5
Weekend visit	9.5	9.5
House C		
Double date	6.5	8.5
Friend after college	7.0	7.0
Roommate	9.0	9.0
Weekend visit	7.0	7.0

than dealing with many partitions and choosing one arbitrarily.[11] The second is that the separate partitions have been responsive to idiosyncratic features of the single relational matrices. This is apparent in Fig. 7 where some of the inconsistencies appear in different places in the separate networks. The partitioning algorithm responded to the different locations so that the blockmodels were over fitted by being responsive to minor differences in similar relations.

Using the approach described here, to increase the chances for delineating a unique partition, presumes that the observed indicators of the underlying affect relation do conform to a single factor model which always requires empirical confirmation. Another reason for the increased likelihood of a unique partition is that the valued network has more information (Žiberna, 2007) and is reflected in the table shown in footnote 6. Valued data provide more information than binary data. As a thought experiment, comparing data collected when the permitted values are (−1, 0, 1) with data collected with permitted scale values, for example (−5, −4, . . ., −1, 0, 1, . . ., −4, −5), suggests that the latter has more discriminating information to exploit in analyzing these data. Any dichotomization of the valued data loses information. The valued data considered here came from summing a set of binary-signed relations and represents another way of obtaining valued data, albeit with a different foundation. Given that scales with more values run the risk of having lowered reliability, it is not clear that summed the binary relations is an inferior approach. This is a measurement issue to resolve in future work. The 'increased information leading to a single optimal partition' argument may require qualification because the summed relation using all four indicators for the T4 Sampson data has two equally well fitting partitions. This is due more to the sanction variable being sufficiently different from the other three indicators. If all of the indicators do tap the same underlying relation then the 'increased information' argument for reducing multiple partitions has great merit and serves as an additional rationale for using multiple indicators.

While it is clear that 'stacking' signed relations, or the separate positive and negative relations, and computing some (dis)similarity measure is not a viable option for signed data, it is possible to simultaneously partition multiple signed relations. Suppose that there are R relations. If the rth relation were partitioned by itself then the criterion function is $\Phi_{\mathrm{r}}(C) = \alpha\mathcal{P}_{\mathrm{r}} + (1-\alpha)\mathcal{N}_{\mathrm{r}}$ where $\mathcal{P}_{\mathrm{r}}$ is the number of positive ties where they should not be for the rth relation and $\mathcal{N}_{\mathrm{r}}$ is the number of incorrectly located negative ties for that relation. The R relations could be partitioned simultaneously by defining the criterion function $\Phi(C) = \Sigma_{\mathrm{r}}\Phi_{\mathrm{r}}(C)$ and minimizing $\Phi(C)$ across all relations with the same relocation algorithm. This merits exploration as an alternative approach.

Given the results in Section 3 some suggestions follow. The *first* is that given reliable multiple indicators of an underlying social relation, it is preferable to analyze the summed variable rather than analyzing separate indicator relations. It follows that it is desirable to collect data for multiple relations as indicators of a common underlying affect relation. The *second* suggestion is that the structural balance state of a network, at a given point of time, *need not hold to the same extent over the whole signed network*. At face value, this is an obvious result even though expressions of a "movement towards balance" for a signed network are usually made for the whole network. Parts of a signed network may be organized according to structural balance theory while other parts organized by some other structural principle(s). That the level of balance can vary across parts of the network can be an important clue for understanding how the social processes operate over time to generate signed networks and can help identify additional mechanisms to structural balance. Having a well-founded blockmodel allows us to focus on those other parts of the network to discern possible alternative organizing principle(s).

Because all of the 'consistency theories' posit some movement towards consistency over time, with structural balance theory one variant, these theories cannot be tested with cross-sectional data. This leads to a *third* suggestion: collecting multiple indicator relational data over time increases the chances of having adequate tests of structural balance theory (or any variant of consistency theories) and identifying additional principles for the organization of signed relations. While the House data are static, the Sampson data on relations are temporal. The prior analyses of the Sampson data by Doreian and Mrvar (1996) showed movement towards balance over time but left an element of ambiguity with two equally well fitting partitions at the final time point (T4). It would appear that the mechanism of change was one of increasing polarization over an exiting partition into three clusters rather than any restructuring of the factions.

Few signed relational data sets reveal groups that are perfectly balanced. This may be due to a mismatch between the data collection design and the dynamic nature of balance theory. When data are available only for a single time point, these data were collected for some unknown stage of a balance process, assuming that one is operative. For the House data, the women had been living together for 4 months. Without knowing the time scale of the process these data cannot speak to testing the process even though it is reasonable to assume that women knew each other well. A more likely reason for not observing a perfectly balanced state is that balance is not the only process that is operative and the (singular) 'balance process' is really a set of multiple processes (Doreian and Krackhardt, 2001). Variables such as cohesion, solidarity, differential popularity (and dislike) or contentiousness may also be relevant and, if so, needs to be considered closely. Lemann and Solomon, using their own measures, argued that House A was the least cohesive of the three houses. This is consistent with the structural features found here: having a signed network being closer to balance. Also needed for a better understanding of the processes involved are data on the individuals themselves. Lemann and Solomon did collect data on personality traits of the women in the three houses but did not report them in detail. Individual characteristics, more generally, and sociometric structures co-evolve. So a *fourth* suggestion is that it will be necessary to collect data for individual level variables as well as dyadic data for distinct social relations *over time* in order to study the co-evolving dynamic change of sociometric structure and actors.

Marsden (2006) is correct in observing that the early pioneering blockmodeling work considered multiple relations and that the generalized blockmodeling approach proposed by Doreian et al. (2005), for all of its innovations, focused primarily on single relations while acknowledging the need to consider multiple relations. A modest step in this direction is taken here for signed networks where the multiple relations have been viewed as indicators of some underlying affect dimension. For the house data, a common partition, obtained from the summed variable, can be imposed on each of the measured variables and these can then be examined when that has substantive interest in its own right. Such a representation, as done in Fig. 7, forms a kind of three-dimensional blockmodel. This data structure differs to the multiple relational data provided by Roethlisberger and Dickson (1939) where the relations could *not* be viewed as being indicators of

[11] For the two occasions when the separate optimized values are the same as the measure of fit under the imposed partition, the partition established from using the summed relation is one of the identified partitions for the relation by itself.

one underlying dimension. Breiger et al. (1975) used these data in their influential paper on blockmodeling. How to blockmodel such three-dimensional network structures is less clear even though the 'stacking approach' to blockmodeling can be viewed, in retrospect, as an attempt to work with multiple indicators. Another approach is to view the set of relations as forming a multi-graph where vertices have multiple arcs.[12] This produces the same results as explicitly summing the relations for House A because there were no cases of a woman selecting another positively on one relation and negatively on another. I suspect that the alternative strategy using $\Phi(C) = \Sigma_r \Phi_r(C)$ would yield the same results in this case. For the other houses using the multi-graph approach produces the same partitions only with slightly different values for the criterion function. This could also be the case when using $\Phi(C) = \Sigma_r \Phi_r(C)$ but as the discrepancies between the observed relations increase, the three approaches to partitioning could be impacted in different ways—yet another area requiring further research.

While the data examined here were signed, it will be beneficial to use a multiple indicator approach for unsigned relations when the observed variables are genuine indicators of some underlying dimension. For studying multiple relations within the blockmodeling approach, there appear to be three directions in which we could head. The most difficult is to replace 'stacking' multiple relations by full three-dimensional blockmodeling. The block types suggested by Doreian et al. (2005) will become three-dimensional 'boxes' rather than 'blocks' with the direct fitting of three-dimensional blockmodels for both signed and unsigned relations.[13] For signed relations, the boxes will be positive or negative. Winship and Mandel (1983) suggest such an approach, albeit in a different form, for unsigned relations. A simpler alternative is to fit three-dimensional blockmodels using an indirect approach. Some steps in this direction are taken by Batagelj et al. (2007) using structural equivalence for unsigned relations. Using multiple indicators for a restricted data structure has the merit of being simple. It is likely that for the first two approaches, a multiple indicator approach for the measurement of social relations will be important to ensure reliable relational data for blockmodeling efforts for discerning social structure, describing those structures and understanding the structural dynamics generating them. No doubt, this will include the specification of different blockmodels for signed networks depending on the location of positive and negative ties that are treated as inconsistencies within a strict structural balance approach.

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[12] This can be done in pajek with the command: Net/Transform/Multiple Relations/Change Relation Number-Label (and then selecting the relations to be included). The multiple lines can be combined using the command Net/Transform/Remove multiple lines/Sum Values.

[13] Doing this will require complex programming.